THE

MATHEMATICAL

GAZETTE

EDITED BY
W. J. GREENSTREET, M.A.

WITH THE CO-OPERATION OF
F. S. MACAULAY, M.A., D.Sc., F.R.S.
AND
Prof. E. T. WHITTAKER, Sc.D., F.R.S.

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OUR TWO-HUNDREDTH NUMBER.

This special number gives us a welcome opportunity of inserting the portraits of some of those who have rendered long and valuable service to the Mathematical Association—as officers, as contributors to the *Gazette*, or in both capacities.

First and foremost is that of **Charles Pendlebury**, M.A., F.R.A.S., who has been our *Hon. Secretary* since 1886. It is needless to dwell upon his devotion to the interests of the Association during the past forty-three years. Suffice it to say that as far as the progress of the Association can be said to be due to any single individual, it is to Charles Pendlebury that we are most of all indebted.

During the last seventeen years he has been ably assisted in the work of his office by **Miss Margaret Punnett**, B.A., who has, since 1912, shared the duties of *Hon. Secretary*. She rendered invaluable assistance as Secretary of the Girls' School Committee which in 1916 drew up the Report on *Elementary Mathematics in Girls' Schools*, and again as Secretary to the Sub-Committee which prepared the Report of 1923 on *The Teaching of Geometry in Schools*. To her hands have been entrusted the relations of the Association with the representatives of the Girls' Schools.

Edward M. Langley, M.A., was our *Hon. Secretary* 1885-1893. He first contributed to General Reports forty-two years ago, and was *Hon. Editor* of the *Gazette* in its quarto form, 1894-1895. Until the last few years he has been a valued and very frequent contributor to our pages.

PROFESSOR **Alfred Lodge**, M.A., contributed to the first number of the *Mathematical Gazette*, then, April 1894, in quarto form, and his name has been rarely absent for long from its pages. The present number contains a characteristic article from his pen. He served as *Hon. Treasurer* from 1891 to 1896, and he was our *President* in 1897 and 1898.

The REV. **John James Milne**, M.A., contributed to the third (quarto) number of the *Gazette*, December 1894, and to the *General Report* issued by the Association in 1887. His contributions to the *Gazette* have been mainly on Geometrical Conics. The present number contains a short contribution connected with his favourite subject. For twenty years he served as *Hon. Treasurer* (1879-1899), and shared the duties of the *Hon. Secretary* in 1896.

William John Greenstreet, M.A., contributed to the second (quarto) number of the *Gazette*, July 1894. He has been *Editor* of the *Gazette* for thirty years (1898-   ).

Francis Sowerby Macaulay, M.A., D.Sc., F.R.S., was *Hon. Editor* of the *Gazette* in 1896 and 1897. From 1898 he also co-operated with the present Editor in its production. He was *Hon. Auditor* for ten years (1917-1926). His first contribution to the *Gazette* was to vol. i., April 1896.

Professor **Edmund Taylor Whittaker**, M.A., Sc.D., F.R.S., was our *President* in 1919 and 1920. His first contribution to the *Gazette* was in December 1904, and he has co-operated with the Editor since 1900.

[The Editor takes this opportunity of publicly thanking his co-operators for the invaluable assistance that has always been at his call. Having two such "guides, philosophers, and friends" at his back has made his work far easier than it might otherwise have been.]

Frederick William Hill, M.A., served on the Editorial Committee of the *Gazette* in 1896 and 1897; he acted as co-operator in 1898. For twenty-nine years he has served as *Hon. Treasurer* (1900-).

The late Sir **Alfred George Greenhill**, F.R.S., was our *President* in 1913 and 1914. He was a conspicuous figure at our Annual and at the London Branch meetings, and a frequent contributor to the *Gazette* of articles and reviews on the subjects in which he was especially interested. It was impossible to obtain any photograph of Sir George except that in which he is one of the three Johnians heading the Tripos list of 1870. It was taken shortly after the results of the Tripos were announced: Senior Wrangler, Richard Pendlebury (the brother of our Secretary); second, A. G. Greenhill; third, E. Levett, brother of one of our Founders—Rawdon Levett.

Arthur Warry Siddons, M.A., contributed to the *Gazette* as far back as vol. ii. 1901. He will be best remembered for the sterling work which he, together with the late Charles Godfrey, *arcades ambo*, put from 1901 to 1910 into the Reports of Committees on the Teaching of Mathematics. On most of these he served as Secretary, and for years he rarely missed an attendance, rivalling in this respect the remarkable record of our *Hon. Secretary*.

Professor **Eric Harold Neville**, M.A., F.R.A.S., became *Hon. Librarian* in 1923. He threw himself into the work of his office with characteristic ardour. He has written a catalogue of real bibliographical value, and has organised the administration of the Library so as to make it much more useful to members than was previously possible. He has provided for the time a local habitation for the books themselves. He has carried to a fine art the delicate negotiations so often necessary in borrowing from a Member, and even from Institutions or from individuals who are not Members, such volumes as the Library has not been able to supply. He acted as Deputy *Editor* for the best part of 1927. His work as Chairman of the Sub-Committee which prepared the Report of 1923 on *The Teaching of Geometry* will not be readily forgotten.

This does not of course exhaust the list of those who are still with us, who have done yeoman's service on Committees, or who have for more than a quarter of a century been contributors to the *Gazette*. Foremost among these is the Rev. James Maurice Wilson, D.D., Canon of Worcester. He was one of the first two Vice-Presidents, in 1871, and took a notable part in the discussions at the meetings for many years. He was *President* in 1921.

We append a list, with dates of early contributions:

- 1896. Sir Joseph Larmor, F.R.S., *President*, 1895-1896.
- 1899. W. F. Beard, M.A.
- 1901. W. J. Dobbs, M.A.; W. H. Macaulay, M.A.; the Rev. E. M. Radford, M.A.
- 1903. Professor E. B. Elliott, F.R.S.; Professor G. H. Hardy, F.R.S.; Professor G. N. Watson, F.R.S.
- 1904. The Right Rev. the Bishop of Birmingham, F.R.S.; Professor H. Hilton, D.Sc.; Major P. A. MacMahon, F.R.S.; D. Mair, M.A.; C. E. Youngman, M.A.

The other members dated 1871 are: Mr. Alfred Allinson Bourne, M.A.; the Rev. Egerton F. M. MacCarthy, M.A.; and Sir Thomas Muir, C.M.G., F.R.S.

Of these veterans, Canon Wilson and the Rev. E. F. M. MacCarthy are the only living original members of the A.I.G.T.

NOTES ON SOME MINOR ENGLISH MATHEMATICAL SERIALS.

By Raymond Clare Archibald, M.A., Ph.D., Hon. LL.D., Professor of Mathematics, Brown University, Providence, Rhode Island, U.S.A.

While major mathematical problems were discussed already in the seventeenth century in such serial publications as the *Philosophical Transactions* (1665) and *Acta Eruditorum* (1682), it was not till the early part of the eighteenth century that serials containing elementary problems of wide appeal commenced to appear. The first of these seems to have been the *Lady's Diary*, started in London in 1704, and "designed for the sole use of the female sex". It had an immediate success, and continued to appear in various forms for 168 consecutive years. Of the second, *Delights for the Ingenious*, there were only eight numbers, in 1711. The third publication of the kind was possibly *Kunstfrüchte* 1. *Sammlung* (1723), a publication (I have not seen it) of the Hamburg mathematical society, founded thirty-three years before. The *Jahrbriefe* of this same society were published irregularly during the 140 years 1732-1871. For the fifth and sixth publications we come back to England, *Miscellaneæ Curiosæ* (York, 1734-35) and *Gentleman's Diary*, started in 1741 and continued for a century before its union with the *Ladies' Diary*. Then followed five other English serials before Holland's *Mathematische Liefhebberye* (Purmerende, 1754), issued annually for seventeen years. During the next 175 years the number of these minor publications became very large. It is my purpose to bring together brief notes on minor English serials and their editors of the past two and a quarter centuries, and to indicate where more information regarding them may be found. It will not be possible, within the limits of this article, to indicate more than very occasionally anything of the contents (often rich and varied) of such serials, many results appearing in their pages for the first time. Problems and their solutions usually occupied the greater part of the space, and most of the prominent English mathematicians of the time were contributors. While some may incline to frown upon such mathematical occupations, it may be recalled that "Sätze und Aufgaben" were to be found in such an exalted source as Crelle's *Journal*, so recently as 1858.

Regarding such minor serials Playfair wrote as follows in 1808: "A certain degree of mathematical science, and indeed no inconsiderable degree, is perhaps more widely diffused in England, than in any other country in the world. The Ladies' Diary, with several other periodicals and popular publications of the same kind, are the best proofs of this assertion. In these, many curious problems, not of the highest order indeed, but still having a considerable degree of difficulty, and far beyond the mere elements of science, are often to be met with; and the great number of ingenious men who take a share in proposing, and answering these questions, whom one has never heard of any where else, is not a little surprising. Nothing of the same kind, we believe, is to be found in any other country. . . . The geometrical part . . . has always been conducted in a superior style; the problems proposed have tended to awaken curiosity, and the solutions to convey instruction, in a much better manner than is always to be found in more splendid publications."

By the middle of the century the standards of excellence had been greatly raised, so that it is not surprising to find T. P. Kirkman (whose name will always be associated with the "problem of fifteen school girls" *) writing as follows in 1849: "I confess it to be my belief, from a limited observation of graduates and non-graduates, that when the difference between prizes awarded by the authorities on either side is considered, an incomparably greater share of the glory of kindling and cherishing a pure and lasting love of mathematical

* Of course his fame will rest on other things; see Macfarlane, *Ten British Mathematicians* 1916, pp. 122-133.

science in *men* as well as *boys*, must be attributed to the immortal Lady Dia, than to all the universities and colleges of these kingdoms put together, to all our Lyceums, Athenæums, and Philosophical Societies, and to all our Imperial Boards of peace and war."

When known, the location of one copy of each serial, in England and the United States, has been indicated. Apart from such citation of original documents, some general references to the literature of our subject are given below. More particular references are to be found in the body of the article.

W. A. Abram, " Memorial of the late T. T. Wilkinson, F.R.A.S., of Burnley ", *Trans. of the Historic Society of Lancashire and Cheshire*, ser. 3, vol. 4, 1876, pp. 77-94 ; Wilkinson's autobiography occupies about ten of these pages.

M. Brierley, 1. " The earlier English mathematicians ", *Papers of the Manchester Literary Club*, Manchester, vol. 2, 1875-76, pp. 175-177.

—— 2. " Lancashire mathematicians ", *Papers of the Manchester Literary Club*, vol. 4, 1878, pp. 7-30.

J. S. Mackay (1843-1914), " Notice sur le journalisme mathématique en Angleterre ", *Assoc. Fr. pour l'Avanc. d. Sc., Compte rendu*, 1893, part 2, pp. 303-308.

T. T. Wilkinson (1815-75), 1. " Mathematical periodicals " [50 articles, regarding 30 periodicals], *Mechanics Mag.* (*M.M.*) vols. 54-59, 1848-53 ; written on the suggestion of T. S. Davies.

—— 2. " On the origin and progress of the study of geometry in Lancashire ", *Notes and Queries*, vol. 2, 1850, pp. 57-60. See also [T. S. Davies] (Pen-and-Ink, *pseud.*), pp. 8, 436-438.

—— 3. " English mathematical literature ", *Westminster . . . Review*, vol. 55, 1851, Amer. ed. pp. 35-42, English ed. pp. 70-83 [anonymous]. Reprinted with alterations in *Educational Times*, No. 42, vol. 4, 1851.

—— 4. " Lancashire geometers and their writings ", *Memoirs Lit. and Philos. So. Manchester*, ser. 2, vol. 11, 1854, pp. 123-157 + 1 plate.

—— 5. " Notae mathematicae ", I-XI, *M.M.* vols. 60-67, 1854-57.

—— 6. " The ancient geometrical analysis illustrated from the writings of the Lancashire geometers ", *Transactions of the Historic Society of Lancashire and Cheshire*, vol. 8, 1856, pp. 75-92.

—— 7. " Biographical notices of some Liverpool mathematicians ", *Trans. Hist. So. Lancashire and Cheshire*, n.s. vol. 2, 1862, pp. 29-40 + 1 plate.

—— 8. " Isaac Rowbottom and the Derbyshire mathematicians ", *Reliquary*, vol. 11, 1871, pp. 201-202.

In the following serial list, which is abridged on account of space limitations, there are several titles not mentioned in any previous list. Where no place of publication is mentioned it is to be inferred that this place is London. Some idea of the sizes of the pages is given by measurement in centimetres of the serials inspected.

1. *The Lady's Diary : or, the Woman's Almanac,** annual, 1704-1840 with varying titles, that of the last being, *The Ladies Diary*. (Sizes 10×16 cm. to $11 \times 17{\cdot}8$ cm. trimmed.) Continued, in amalgamation with *The Gentleman's Diary* as *The Lady's and Gentleman's Diary* (No. **3**). The British Museum (B.M.) set lacks the issues for 1704, 1705, part of that for 1792, and part of the last leaf for 1821. The Brown University (B.U.) set lacks 1704-09, 1711-12, 1714-15, 1725-26, 1728-39. Is there any public library where the issues for

* Wilkinson, (i) *M.M.* vol. 48, 1848, pp. 56-57 ; vol. 51, 1849, p. 484 : (ii) *Educational Times*, vol. 17, 1864, pp. 82-83, 176-178, 1865, pp. 270-271 ; vol. 18, 1865, pp. 55-57, 128-129 ; vol. 19, 1866, pp. 9-10, 197-198 ; T. Leybourn, *Mathem. Questions . . . Ladies' Diary*, vol. 1, 1817, pp. v-xi, see No. **1**. **4** ; Playfair, *Edinburgh Rev.* vol. 11, 1808, p. 282 ; T. P. Kirkman, *Lady's and Gentleman's Diary*, 1850, p. 85 ; Wilkinson, " An account of the early mathematical and philosophical writings of Dr. Dalton," *Mem. Lit. Phil. So. Manchester*, ser. 2, vol. 12, 1855, pp. 2-25.

1704 and 1705 have been preserved? The editors of the *Diary* were as follows:

1704-1713, John Tipper,* d. 1713, founder of the *Diary*, of *Delights for the Ingenious* (No. **4**), and of *Great Britain's Diary* (1710-28), non-mathematical.

1714-1744, Henry Beighton,† d. 1743, eminent surveyor and engineer, fellow of the Royal Society.

1745-1753, Robert Heath,‡ d. 1779, also editor and founder of: (*a*) *The Palladium* (No. **8**); (*b*) *The Ladies Chronologer* (No. **11**); and (*c*) *The Ladies Philosopher* (No. **10**). He was dismissed from office because he used for his own periodicals material sent for publication in the *Lady's Diary*, and also because he used the *Diary* as a vehicle of personal abuse.

1754-1760, Thomas Simpson,§ d. 1761, professor of mathematics at the Royal Military Academy, Woolwich, from 1743, fellow of the Royal Society, and author of at least ten mathematical works.

1761-1773, Edward Rollinson, d. 1773, editor of the *Mathematician* (No. **6**).

1774-1817, Charles Hutton,|| 1737-1823, professor at the Royal Military Academy, Woolwich, 1773-1807, fellow of the Royal Society, author of many books and memoirs. Editor also of *Diarian Miscellany* (No. **1.1**), and *Ladies Diary Suppl.* (No. **1.3**).

1818-1840, Olinthus Gregory,¶ 1774-1841, professor of mathematics at the Royal Military Academy, Woolwich, 1807-1838, editor of the *Gentleman's Diary*, 1804-19, and author of many works on biography, theology, astronomy, and mathematics.

While these were the nominal editors there were others who assisted from time to time; for example, it is said that during the latter years of Beighton's administration Anthony Thacker (died in 1744; see **2.1**) was the editor of the mathematical part ** while Beighton's wife edited the enigmas.

The first questions which were really mathematical appeared in the *Diary* for 1707. Over 1800 such questions and their solutions were published.

1.1. *The Diarian Miscellany; consisting of all the useful and entertaining parts, both mathematical and poetical, extracted from the Ladies' Diary from the beginning of that work in the year* 1704 *down to the end of the year* 1773. *With many additional Solutions and Improvements.*†† This edition by Charles Hutton was issued originally in fourteen parts,‡‡ July 1, 1771–July 1, 1775. The parts

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* *Dict. Nat. Biography* (*D.N.B.*). Five letters of Tipper (Harl. MS. 3782) giving interesting particulars about the starting and conduct of the *Diary* are printed in H. Ellis, *Original Letters of Eminent Literary Men*, London, 1843 (*Camden So. Publ.* vol. 23, pp. 304-315). Of the second *Diary* the 4000 copies printed were all sold by "New-Years-tide."

† C. Hutton, *Philos. and Mathem. Dict.* 2nd ed. 1815; *D.N.B.*

‡ Leybourn, *Mathem. Questions ... Ladies Diary*, vol. 1, pp. viii-ix; *D.N.B.*; Wilkinson, *M.M.* vol. 67, 1857, p. 607.

§ Hutton, *Philos. and Mathem. Dict.* 2nd ed. 1815; *D.N.B.*

|| *D.N.B.*; *Encycl. Brit.* 11th ed. For a correction of the former in its reference to Hutton's son, see a note by P. J. Anderson in *Notes and Queries*, ser. 11, vol. 2, 1910, p. 347. There is a portrait in *European Mag.* vol. 83, 1823, p. 483.

¶ *D.N.B.*; *Encycl. Brit.* 11th ed.; poems in his memory, *Lady's and Gentleman's Diary* for 1842, pp. 19-20.

** In Thacker's *Miscellany of Mathematical Problems*, vol. 1, Birmingham, 1743, the author styles himself "Teacher of Mathematicks at the Birmingham Free-School. And the Author of the Ladies Diary," which lends support to this statement.

†† Wilkinson, *M.M.* vol. 61, 1854, p. 244, and *Reliquary*, vol. 11, 1871, p. 202; *Ladies Diary*, 1771, p. 46.

‡‡ Part *1*, July 1, 1771, pp. 1-60 math. (pp. 1-120 rebuses, ænigmas); *2*, Nov. 1, pp. 61-120 (pp. 121-144); *3*, Feb. 1, 1772, pp. 121-180 (pp. 145-168); *4*, May 1, pp. 181-252 (pp. 169-240); *5*, Aug. 1, pp. 253-324 (pp. 241-324); *6*, Nov. 1, pp. 325-364 end of math. vol. 1; *7*, March 1, 1773, pp. 1-140 math. vol. 2 (pp. 325-384); *8*, July 1, pp. 141-248 (pp. 385-396 end of poetry, etc. vol. 1, and pp. 1-48, vol. 2); *9*, Nov. 1, pp. 249-392 end of math. vol. 2 (pp. 49-120); *10*, March 1, 1774, pp. 1-72 (pp. 121-168); *11*, July 1, pp. 73-168 (pp. 169-216); *12*, Nov. 1, pp. 169-288 (pp. 217-288); *13*, March 1, 1775, pp. 289-372 (pp. 289-348); *14*, July 1, pp. 373-424 end of math. vol. 3 (pp. 349-364 end of poetry, vol. 2). See *Notes and Queries*, ser. 11, vol. 3, 1911, pp. 252-253.

were assembled in five volumes, the first three mathematical, with title-pages dated London, 1775: I, 370 pp.; II, 394 pp.; III, 426 pp.; IV, 398 pp.; V, 366 pp. (10·4 × 17·2 cm.). Volume 3 contains an index to all proposers and solvers of mathematical problems. In volume 5 is given a list of all enigmas 1704-1773. Hutton's own copy, in parts, with their original covers and MS. notes, is in B.M.; copies of the 5-vol. work are in B.U.

Lowndes states that *Miscellanea Mathematica* (No. **19**) "forms the sixth and concluding volume of the preceding work" [*The Diarian Miscellany*].* On page 3 of the cover of No. IV (May 1, 1772) of *The Gentleman's and Lady's Miscellany* (that is *The Diarian Miscellany*) is the following which may be quoted in corroboration: "it may be observed that it is intended to continue the work till we overtake the original Diary which it is supposed will be in the year 1774; by which time there will be completed 2 or 3 volumes of the Mathematics, and the same of the Poetry, besides a small one of the New Miscellany all of which must be bound separately, as they are paged, for which purpose in due time will be given proper Titles, Prefaces, Indexes, etc."

1.2. *The Diarian Repository; or, Mathematical Register: containing a complete Collection of all the Mathematical Questions which have been published in The Ladies Diary from the Commencement of that work in* 1704 *to the year* 1760 *together with solutions fully investigated according to the latest Improvements* . . . By a Society of Mathematicians. (16·5 × 22·7 cm.) 1774, 720 pp. Copies in B.U. and B.M. This was published in thirty numbers beginning between July 1 and Nov. 1, 1771, and was, apparently, designed by the editor, Samuel Clark (=Society of Mathematicians), as a rival to Hutton's *Diarian Miscellany* (No. **1.1**) started a little while before. This, and other actions of Clark, provoked Hutton not a little.† In 1812 he referred to Clark's *Repository* as follows: "From the many gross errors, and the numerous omissions and absurdities in that 'Repository of Errors', as it was commonly called, it soon fell under the necessity of coming to a sudden and premature end." Clark died in 1784, with few to mourn his demise.‡ See also under Nos. **15** and **17**.

1.3. *A Supplement to the Ladies Diary*, 1788-1790; *A Companion, or Supplement to the Ladies Diary*, 1791; *The Diary Companion being a Supplement to the Ladies Diary*, 1792-1806, nineteen annual issues of 48 pp. each except the first number of 32 pp.§ By the *Diary* author, that is, C. Hutton. (10·5 × 17·3 cm.) Copies in B.M. and B.U.

1.4. *The Mathematical Questions proposed in The Ladies Diary, and their Original Answers together with some new Solutions, from its Commencement in the year* 1704 *to* 1816. *In four volumes.* By Thomas Leybourn,‖ 1817. Vol. 1, 426 pp.; vol. 2, 418 pp.; vol. 3, 402 pp.; vol. 4, 442 pp. (13·5 × 20·8 cm.) Copies in B.M. and B.U. An admirable edition with an index of the names of the persons who proposed and answered the questions; there is also a subject-index for the questions. Hutton's edition (No. **1.1**) includes some astronomical questions not given in Leybourn's and gives the original enunciations. Leybourn sometimes substitutes brief prose for poetry in this connection. See comment under No. **25**.

2. *The Gentleman's Diary, or the Mathematical Repository: An Almanack*

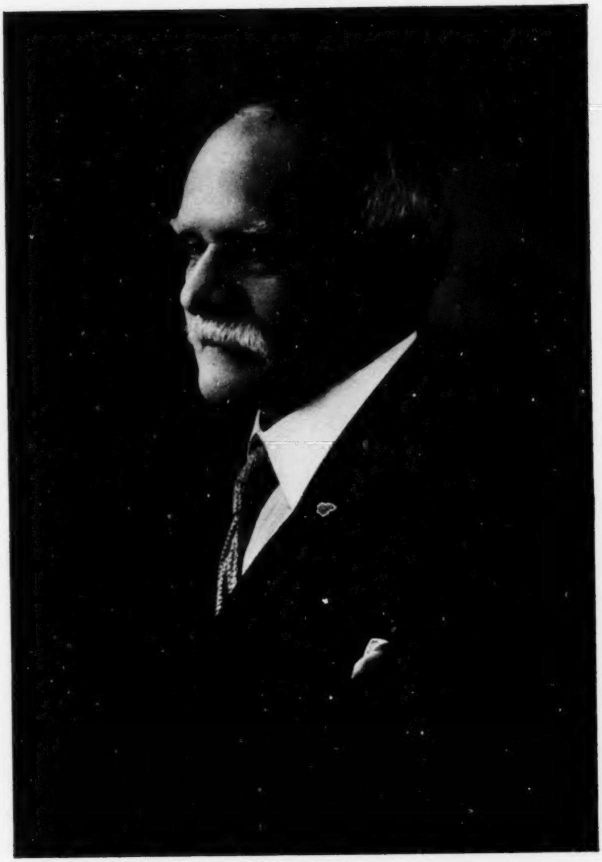
* P. J. Anderson, *Notes and Queries*, ser. 11, vol. 2, 1910, p. 347; see also vol. 3, 1911, p. 253.

† Hutton, *Diarian Miscellany*, B.M. copy, covers of Nos. II, VIII-XIV; Hutton, *Tracts on Mathem. and Philos. Subjects*, vol. 3, 1812, pp. 381-2; Leybourn, *Mathem. Questions . . . Ladies' Diary*, vol. 1, p. 5; *Notes and Queries*, ser. 11, vol. 3, 1911, p. 252.

‡ Compare Wilkinson, *M.M.* vol. 61, 1854, pp. 244-246.

§ Wilkinson, *M.M.* vol. 51, 1849, pp. 484-486.

‖ There is an inadequate and inaccurate sketch of Leybourn (1770-1840), by Charles Platts, in *D.N.B.* For most of the last forty years of his life he was teacher of Mathematics in the Royal Military College, Sandhurst. He was also editor of *The Gentleman's Diary* (No. **2**), *The Mathematical Repository* (No. **25**). See also *Math. Gazette*, vol. 6, 1912, p. 297.



C. Pendlebury.

*... containing many useful and entertaining Particulars peculiarly adapted to the ingenious Gentlemen engaged in the Study and Practice of the Mathematics.** Annual, 1741-1840, 48 pp. to nearly every issue; there were appendices of twenty-four pages containing mathematical papers in issues for the years 1835-39. (11 × 17·5 cm.) The B.M. set lacks pp. 3-14 of each of the issues 1791, 1792, 1810. The B.U. set lacks 1741, and the first sixteen pages (almanacs) of each of the years 1752-77. Continued, in amalgamation with the *Ladies' Diary*, as *The Lady's and Gentleman's Diary* (No. **3**). Regarding the union of the diaries there were many poems (*e.g. L. and G. Diary* for 1842; one of them was "Auspicious nuptials" by Charlotte Caroline Richardson (1777-1853, *D.N.B.*), author of a volume of poems, and for nearly forty years a contributor of poetical material to the *Ladies' Diary*; another by John Hope was entitled: "Elegy on the demise of the Gentleman's Diary"; and a third by Noah Wilmot began

"Though unions now are all the rage,
Who would have thought that Lady Di
Would at her patriarchal age
Have listen'd to a lover's sigh?"

The first planners of the *Diary* were John Badder, George Ingman, Thomas Peat, Wm. Whitehead, Anthony Thacker, Wm. Butcher, Thomas Mather, and J. Granger. It was continued chiefly by the first three from 1741 to 1756, when Badder died and Ingman withdrew. Peat continued as editor from 1757 till his death in 1780. Rev. Charles Wildbore † (d. 1802) was the editor of the issues for 1781-1803, Olinthus Gregory (d. 1841) for 1804-19, and Thomas Leybourn ‡ (d. 1840) for 1820-40. See also under No. **26**.

2. 1. *A Collection of Mathematical Problems and Aenigmas; or A Supplement to the Gentleman's Diary*, (1) for the years 1741 and 1742, 50 pp., 1743; (2) for the year 1743, 52 pp., 1744; (3) for the year 1744, 46 pp., 1745. (10 × 16·5 cm.) Copies in B.U. The first two numbers contained overflow material from the *Gentleman's Diaries* and were edited by the *Diary* authors, of whom Thomas Peat, "Writing-master, and Teacher of the Mathematicks, at Nottingham", was one. The third and last number was "by a Society of Gentlemen". For further notes in this connection see under No. **7**.

2. 2. *The Gentleman's Diary or Mathematical Repository.* Davis's edition, vol. 1, 1741-60 [1815-16], 334 pp.; vol. 2, 1761-80 [1817], 308 pp.; vol. 3, 1781-1800, 1814, 338 pp. (11 × 18·4 cm.) The *Supplements* (No. **2.** 1) are included in this edition. There is no index. B.U. and Royal Society have copies.§ The third volume was published first and is the only one that is dated. The dates of the other volumes, each published in two parts, were determined from issues of the *Gentleman's Mathematical Companion* for 1816-18; vol. 1, part 1, was published Nov. 9, 1815. "Improved solutions are added to those questions which were not correctly answered; accompanied by remarks tending to illustrate and advance the science of the Mathematics." See also under No. **28**.

3. *The Lady's and Gentleman's Diary ... designed principally for the amusement and instruction of Students in Mathematics*, published annually (1841-1871). (11 × 17·5 cm.) 72 to 96 pp. each. B.U. set complete; B.M. set lacks the

* C. Wildbore, *Gentleman's Diary*, 1781, preface, pp. 2-4, reprinted in Davis's edition, vol. 3; O. Gregory, *Gentleman's Diary*, 1804, p. 2; Wilkinson, *M.M.* vol. 48, 1848, p. 57.

† *Gentleman's Mag.* vol. 72, part 1, 1802, pp. 1075, 1221. Allibone, *Critical Dictionary of Engl. Lit.* vol. 3, 1871. Wildbore was pastor at Sulney, Notts., for over thirty years, and was a contributor to several minor mathematical serials.

‡ See under **1. 4.**

§ *Cat. Per. Publs. Lib. Royal So. London*, 1912.

issues for 1854, 1856-70. It was edited by W. S. B. Woolhouse * (1809-1893). A valuable publication. See also No. **2**. Many interesting "Mathematical papers", usually filling about twenty-four pages of a number, were published in the issues for 1842-65. In 1844 the editor offered a prize for the solution of the following question: "Determine the number of combinations that can be made out of n symbols, p symbols in each; with this limitation, that no combination of q symbols which may appear in any one of them shall be repeated in any other." This very difficult problem was solved by Kirkman, for the case $p=3$ and $q=2$, in a paper read before the Manchester Literary and Philosophical Society in 1846 and published in *Camb. and Dublin Math. Jl.* vol. 2, 1847 (pp. 191-204). As a chip of this work he published in the *Diary* for 1850 (p. 48) the following famous problem: "Fifteen young ladies in a school walk out three abreast for seven days in succession: it is required to arrange them daily, so that no two shall walk twice [meaning more than once] abreast." A bibliography of this problem including the names of Cayley, Sylvester, Steiner, and B. Peirce, was published in *Messenger of Math.* vol. 41, 1911, pp. 33-6.

4. *Delights for the Ingenius or Monthly Entertainment for the Curious of both Sexes.*† The eight numbers (40 pp. each) of this periodical, founded by John Tipper (see No. **1**), were all published in 1711, and form a volume of 320 pages with a general index (10·2 × 15·8 cm.). The first six numbers, issued at monthly intervals, followed by two more after lapses of three month intervals, treated mathematical questions and enigmas in a somewhat popular manner. There are copies in the American University, Washington, D.C., and in B.M.

5. *Miscellaneæ Curiosæ; or entertainment for the ingenious of both sexes,*‡ York, six numbers, January 1734–September 1735; no number was published Oct.-Dec. 1734. With the second number the title was corrected to *Miscellanea Curiosa.* There are copies in B.M. and B.U. The editor was Thomas Turner. Was he a grandfather of Thomas Turner Tate, editor of the mathematical column in *The York Courant* (No. **39**)?

6. *The Mathematician, containing many Curious Dissertations on the Rise Progress and Improvement of Geometry* § . . . was issued in six numbers: 1, 1745, 8+48 pp.; 2, 1746, pp. 49-123; 3, 1747, pp. 125-188; 4, 1748, pp. 189-260; 5, 1749, pp. 261-340; 6, 1750, pp. 341-401. The title-page of the completed volume is dated 1751. (12 × 20 cm.) B.M. has Nos. 1 (in original covers) and 2 only. B.U. and the Library of the Mathematical Association have the complete volume. It was nominally edited by Edward Rollinson (see No. **1**), assisted by Thomas Simpson (see **1**) and John Turner; but tradition favours Simpson as the real editor. This excellent periodical was believed by some to have been started to offset the bad effects of Heath's *Diary*. See also No. **9**.

7. *Miscellanea Curiosa Mathematica,*‖ fourteen numbers, 1745-1753.

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* *Proc. London Math. So.* vol. 25, 1893, p. 1-4; H. H. Turner, *Mo. Notices R. Astr. So.* vol. 54, 1894, pp. 204-206; G. H. Ryan, *Jl. Inst. Actuaries*, vol. 31, 1895, pp. 362-365. Woolhouse was a deputy superintendent at the Nautical Almanac office 1830-39; his new method of finding the latitude and longitude was characterised as one of the greatest improvements in nautical astronomy of the century. He was the author of (1) a number of volumes, including one on *Measures, Weights and Monies of all Nations*, and (2) many valuable papers in the *Journal* of the Institute of Actuaries. He was also an accomplished musician and editor of classical music. About thirty of his papers are listed in the *Royal Society Catalogue.*

† *D.N.B.* article Tipper, John.

‡ Lawson, *Synopsis of Data for Constr. Triangles*, Rochester, 1773; B.M. *Catalogue*; *Cat. Per. Publs. Lib. Univ. Coll., London*, 1912.

§ Wilkinson, *M.M.* vol. 50, 1849, pp. 5-9; the question of editors is discussed at length. See also Hutton's life of Simpson in Simpson's *Select Exercises*, new ed. London, 1792, p. xviii.

‖ Wilkinson, *M.M.* vol. 53, 1850, pp. 144-148, 193-196; *Newcastle General Mag.* vol. 6, 1753, p. 444; Lawson, *Synopsis of Data for Constr. Triangles*, Rochester, 1773. In the preface to *The Gentleman's Diary* for 1798 the following lines by the editor, Rev. C. Wildbore, occur on the second page: "It is unpleasant to record dirty tricks that men of science have been guilty of; yet, it may be proper on this occasion to observe, that the original authors of this Diary published two annual Supplements to it, and were about to write a third, when they found themselves anticipated by some mathematicians in London, who had surreptitiously

(15·5 × 21 cm.) Volume 1, with 16 + 312 pp. + 9 plates, contains nine numbers,* published 1745-49; of volume 2 there were five numbers only, 186 pp. + 6 plates, the last published by August, 1753. University College, London, and B.U. have the complete work. Well edited by Francis Holliday,† master of the Grammar Free School at Haughton Park, near Retford, Notts. It is in an article of this periodical that one first finds the expression $\sqrt{(R^2 - 2Rr)}$ for the distance between the centres of the inscribed and circumscribed circles of a triangle—a result often attributed to Euler, who gave one something like it nearly twenty years later. Chapple also enunciates the porism: If two circles are such that there exists a triangle inscribed in the first and circumscribed to the second, there exists an infinity of triangles inscribed in the first and circumscribed to the second. One recalls the developments that this theorem received at the hands of Poncelet, Steiner, and Jacobi.

8. *The Palladium or Appendix to the Ladies Diary*, for 1749; *The Gentleman and Lady's Palladium*, for 1750-54, 1758, 1760-2; *The Gentleman and Lady's Palladium and Chronologer*, 1755; *The Gentleman and Lady's Palladium and Diary*, 1756; *The Gentleman and Lady's Diary and Palladium*, 1757; *The Gentleman and Lady's Military Palladium*, 1759; *The Palladium Extraordinary*, 1763; *The Palladium Enlarged*, 1764 + *The Palladium-Supplement*, 1764; *The Palladium of Fame or Annual Miscellany*, 1765; *Fame's Palladium or Annual Miscellany*, 1766; *Fame's Palladium or Annual Miscellany*, 1767; *The British Palladium or Annual Miscellany*,‡ for 1768-79, 1749-1763 (10 × 16·3 cm.); 1764-79 (11 × 17·6 cm.) Complete copies in B.U. and University College, London. Edited by Robert Heath with greater care than had been bestowed upon the *Lady's Diary*, but personal abuse and scurrility abounded. De Morgan characterised Heath as "a person who made noise in his day, and in so doing established a claim to be considered a worthless vagabond". [*Arithmetical Books*, London, 1847, p. 71.) See also under No. **1**.

The first number of the *Palladium* was announced in the preface of the *Ladies Diary* for 1749, *i.e.* four years before his connection with the *Diary* ceased: "Having extended our *scheme* for improving this *Diary*, we are obliged to compose a *Palladium* or appendix to it, of such materials as are connected with, subservient to, and fit to be bound up with the Ladies Diary". He concludes the preface with the following lines:

Hail, happy Ladies! take your Joys complete
Your *Di'ry*, and your *Lov'd Palladium* greet:
No tasteless Art degrades your lively Wit,
Like *Hand* and *Glove*, so charmingly they fit.

inspected the contributors' letters, and published the solutions to the questions; in a quarterly Miscellany then carried on by *Holliday*; who had wrote in the Diary under the name of Gamston Retford". One can verify that the sixteen questions prepared in the second *Supplement* of *The Gentleman's Diary* (No. **2**. 3) were reprinted (except for changes from poetry to prose) by Holliday, with solutions by Samuel Farrer, in the first and second numbers of *Miscellanea Curiosa*. Solutions by others appeared in the third *Supplement* to the *Diary*. That the editors of this *Supplement* did not share Wildbore's opinion is shown by the *Supplement* itself. In commenting on question 2 one finds the following on page 18: "The above was ingeniously answered in Numbers by a Gentleman in the *Miscellanea Curiosa Mathematica*, Number I, to which we refer the Reader; and we shall here, once for all, observe, that our Readers will find most of them solved in the aforesaid *Miscellanea*, but in a quite different Method to what we have given here". But further, there is a not unfriendly paragraph on the *Miscellanea* in the preface, the answer to Question XII (pp. 27-28) *is copied with acknowledgement*, from the *Miscellanea*, as is also (p. 33) "Gamston Retford's Paradox", the third part of a "dissertation" by Gamston Retford is printed on pp. 13-17, and an acknowledged editorial courtesy on the part of the "conductor of the *Miscellanea Curiosa Mathematica*" filled pp. 31-32.

* "On the wrapper of the first number Newton is said to have thus expressed himself with regard to simple but tedious problems:—*They are like the endeavouring to crack, with the teeth, a hard pebble, for the sake of the kernel.*" (R. Copeland, *Cat. Crawford Lib. R. Obs. Edinburgh*, 1890, p. 239.)

† Author of works on: equations and series, *Syntagma Mathesios* (1745), gunnery and fortification (1756, 1774) and *Fluxions* (1777),—according to B.M. *Catalogue*.

‡ Wilkinson, *M.M.* vol. 50, 1849, pp. 466-475; *Cat. Per. Publs. Lib. Univ. Coll. London*, 1912.

To this he afterward added:

> Of *Truth* productive, *Science* to extend,
> A *Foe* to error, and the *Muse's Friend.*

9. *Mathematical Exercises,** London and Wrexham, edited by John Turner, was begun towards the close of 1750, immediately after the discontinuance of the *Mathematician* (No. **6**). (12·2 × 20 cm.) There were six numbers (the last in 1753, when Simpson became editor of the *Ladies Diary*), and they formed a volume of 298 pp. This periodical was set on foot to afford a place for exposing the errors and absurdities of Heath and the *Palladium* (No. **8**). There are copies in B.U. and the Royal Astron. So. (*Catalogue,* 1886). See also under No. **10**.

10. *The Ladies Philosopher,* or *The Lady's Philosopher,* was edited by Robert Heath, and three numbers were published in 1752, 1753, and 1754. It was " designed to comprehend a Variety of curious and interesting Subjects, for general Improvement: And also what there was not room for in the *Lady's Diary,* and this *Palladium* " (*Palladium,* 1753, p. 62). " Emendations of the Lady's Philosopher, No. III " occurs in *Palladium,* 1754, pp. 33-34, which refers to " a blundering solution in . . . No. 4, *Mathematical Exercises* " (No. **9**); and then: " It was expected that R.R.H. in *Lady's Philosopher,* No. III. would have been more severe in his Answer to *Turner,* for who is obliged to take such abuse from a low life (*sic*) Animal, who is not capable of hurting any Body but with his Tongue ". See also *Palladium,* 1754, p. 53, 1755, p. 2, and *D.N.B.,* article Heath, Robert. Where this serial is to be seen, or other facts regarding it, are equally unknown to me.

11. *The Ladies Chronologer,* 1754. (9·8 × 16·3 cm.) A single issue (48 pp.), merged into *The Gentleman and Lady's Palladium and Chronologer,* edited by Robert Heath. The first paragraph of the preface is as follows: " Meeting with unexpected, as undeserved, ill treatment from the Agents of an Honourable Society, after compiling the last *Eight* Ladies Diaries, (from 1744 to 1753) and encreasing (*sic*) the Sale of the *annual* Impression, by many Thousands, (*as the three Printers thereof can witness*) we declined writing the said Diary any longer.—However, to make our ingenious Correspondents amends for their Contributions, we have published them in this our *Ladies Chronologer,* which otherwise must have been lost to the Public: So that it will be seen, whether our *Ladies Chronologer,* or the *Ladies Diary* (as now conducted) is the more useful Production ". Copies in B.U., and University College, London (*Cat. Per. Publs. Lib., London,* 1912).

12. *Miscellaneous Correspondence, containing a variety of Subjects relative to Natural and Civil History, Geography, Mathematics, Poetry, Memoirs of monthly Occurences, Catalogues of new Books,* etc., edited by Benjamin Martin † (1704-82, famous as an instrument maker and general compiler. It was a monthly serial published from January 1755 to December 1763 in four volumes: vol. 1, for 1755 and 1756, title-page dated 1759, pp. 1-460; vol. 2, for 1757 and 1758, pp. 461-964 + 8 pp. index to vols. 1-2, title-page dated 1759; vol. 3, for 1759 and 1760, title-page dated 1764, pp. 1-590; vol. 4, for 1761, 1762, 1763, pp. 591-1146 + 14 pp. index to vols. 3-4, title-page dated 1764. (12·6 × 20·8 cm.) There are copies in B.M. and B.U. There was a mathematical section for problems proposed and solved, and for mathematical articles and correspondence in each number. These four volumes are part of a fourteen volume serial, *General Magazine of Arts and Sciences, Philosophical, Philological,*

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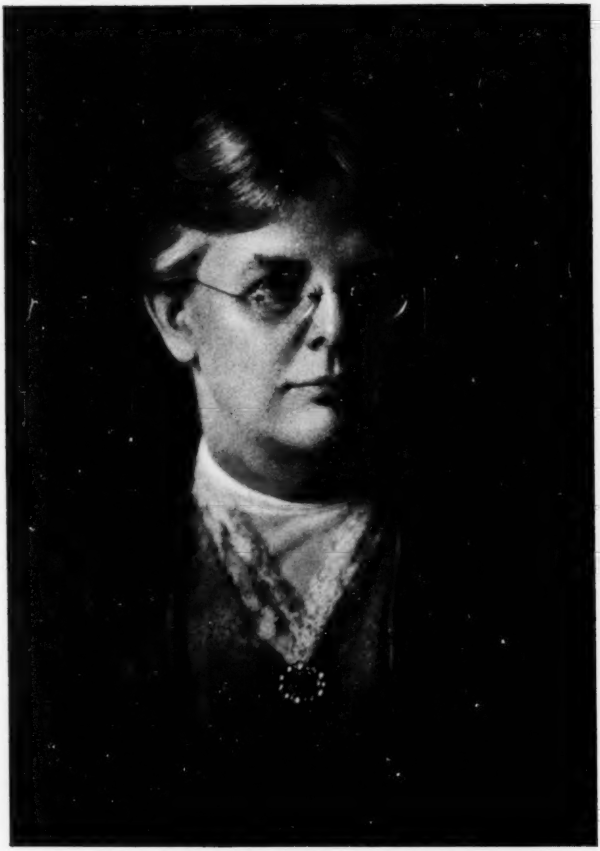
* Wilkinson, *M.M.* vol. 50, 1849, pp. 267-273, and vol. 67, 1857, p. 607. See also T. Simpson, *Select Exercises,* new ed. 1792, p. xviii.

† Lawson's *Synopsis* (*l.c.*); *Gentleman's Mag.* vol. 72, part 1, 1802, pp. 1075, 1221; *D.N.B.*; A. N. Disney and others, *Origin and Development of the Microscope,* London, 1928, pp. 230, 289; in *D.N.B.* Charles Platts makes no references to *Miscellaneous Correspondence,* and his statement regarding the incompleteness of Martin's *Magazine* is wholly erroneous; the complete publication is in B.M.



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MISS PUNNETT.

Mathematical and Mechanical, edited by Martin. Three more of these volumes are of interest to the mathematician: (1) *A Body of Mathematical Institutions or Principles of Science*, 2 vols. 1759, 1764, which may be a mathematical serial (I have not seen it); and (2) *Biographia Philosophica being An Account of the Lives, Writings, and Inventions, of the most eminent Philosophers and Mathematicians who have flourished from the earliest Ages of the World to the present Time*, 1764, 571 pp. + frontispiece portrait of Newton. Copy in B.U. This appears to have been compiled wholly by Martin, but it may, possibly, have been issued in parts in the *General Magazine*.

13. *Imperial Magazine*,* monthly, Jan. 1760–May 1762, 2 vols. and part of a third. Volumes 1 and 3 are in Harvard University and 2 in Library of Congress; imperfect copies of volumes 1 and 2 are in B.M. Beginning in March 1760 there were one to three pages of mathematical questions proposed and solved. There were about forty problems in each of the volumes 1 and 2. John Cowley, author of various geometrical works, and Malachy Hitchins, the astronomer (*D.N.B.*), were contributors; Hitchins was also a contributor to the *Ladies' Diary* for 1761.

14. *The Mathematical Magazine: and Philosophical Repository. Containing a variety of Original Pieces in all parts of Mathematical Science*,† edited by George Witchell and Thomas Moss, was published monthly April-August 1761 (pp. 1-80 + title-page + frontispiece of "Andromeda"). (12·3 × 19·4 cm.) In the issue for June we find (pp. 41-43) "*A curious letter from* Mr. Abraham de Moivre, F.R.S., to Dr. Edmund Halley, F.R.S., *never before published*". The letter is undated and starts as follows:

"Sir,—I believe I have hit upon the right method of demonstrating your theorem for finding the rate of interest in annuities, and of continuing it; if there were occasion for it."

Then follows mathematical discussion ending with an expression (p. 43) for the rate of interest, "which is your very theorem". It may be that this letter is of some value in the history of annuities. Already in the opening sentences of the first edition of his *Annuities upon Lives* (1725) De Moivre spoke highly of Halley's contributions to the basis of the subject of which he treats. As yet I have not located in print (if it was printed) the Halley theorem to which reference is made; if this could be done the letter might possibly be dated approximately. There are complete copies of this periodical in B.U. and University College, London.

15. *Mathematical Transactions and Collections; with a methodical history of Mathematics; by a society of gentlemen.*‡ No. 1, 1762, 32 pp. + plate. (17·9 × 23 cm.) The editor was probably Samuel Clark, author of a work on *The Laws of Chance* published four years before, and editor of *The Diarian Repository* (see under **1**. 2) where he used the pseudonym "Society of Mathematicians." He is the author of the first of four articles which fill the first thirty-one pages. The second article on "A method of determining the time of exhausting any given vessel filled with water or any other fluid" is, apparently, the one referred to by Hutton, *Miscellanea Mathem.* (No. **19**), p. 2, which determines that only one number of the *Transactions* was published. Copies of this are to be found in the library of the Royal Astronomical Soc. (*Catalogue*, 1886) and in B.U. The last page contains six problems for solution.

16. *Miscellanea Scientifica Curiosa*,§ started at the close of 1766 and ended with No. 8 in 1769, 246 p. + 10 plates; there were probably three numbers a

* Lawson, *Synopsis* (*l.c.*).

† Lawson, *Synopsis* (*l.c.*); Wilkinson, *M.M.* vol. 52, 1850, pp. 446-448; *Cat. Per. Publs. Lib. Univ. Coll., London*, 1912.

‡ *Notes and Queries*, ser. 11, vol. 3, 1911, p. 246; vol. 5, 1912, p. 15.

§ Lawson, *Synopsis* (*l.c.*); Wilkinson, *M.M.* vol. 52, 1850, pp. 266-270; A. De Morgan, "Fly leaves of books: Reuben Burrow," *Notes and Queries*, vol. 12, 1855, pp. 142-143; *Cat. Lib. R. Astr. So.* 1886.

year. (18·2 × 23·3 cm.) There is no indication in the publication of who the editors were, and this fact seems to have been first made known in print in a note by De Morgan on Reuben Burrow. The first part of the note is as follows: "This very low-minded mathematician was born in Yorkshire, and was successively a clerk, writing-master, school master, astronomical assistant to Maskelyne, editor of the *Ladies' Diary*, and assistant on the trigonometrical survey in India, where he died in 1791 or 1792. He was a good geometer and an able man, and he left several works in print. He had an excessive hatred of John Green and William Wales, who were successively the astronomers in Cook's voyages, and he had probably been beaten by them in some competition for places. Whenever he bought a work of either, he wrote some scurrility on the fly-leaf, as I have seen in various instances. But in the *Miscellanea Scientifica Curiosa*, of which *both* Wales and Green were editors, he wrote as follows. His copy is in my possession: 'Miscellanea Scientifica Curiosa Or a Balderdash Miscellany of damn'd Stupid Raggamuffin Methodistical Nonsense and Spuability. By two of the most stupid and most dirty of all possible Fools Rogues and Scoundrels, viz[t] John Green A.M. Late Tub Thumper now Soul-driver in Hell and William Wales,—brusher at Christ's Hospital, not only the dirtiest Scoundrel that God ever made, but The dirtiest rascal that he Possibly could make. Amen.' I need not say that Green and Wales * were both respectable men." There is a complete set of this serial in Yale University; No. 1, 26 pp. + 1 plate, is in B.U. According to *Cat. Per. Publs. Lib. Univ. Coll., London*, 1912, this library has Nos. 1-4 (pp. 1-122). There is a complete copy in the library of the Royal Astronomical Society. See also under No. **20**.

17. *Town and Country Magazine.*† In most of the numbers of the first sixteen volumes of this periodical, 1769-84, one to three pages were devoted to astronomical and mathematical problems, solved and unsolved. Samuel Clark was editor of the mathematical section until his death early in 1784. The department was resumed under new management in June. The periodical was continued till 1792, but I have not seen the last eight volumes. Complete sets in B.M. and Library of Congress. See also under Nos. **1.2**, and **15**.

18. *British Oracle; Consisting of Questions, Essays, and Dissertations in Natural Philosophy and the Mathematics,*‡ consists of twelve numbers in one volume (384 pp.) published in 1869-70; the title-page for the volume, bearing the date 1769, seems to have been printed with the first number; one of the communications in the last number is dated March 5, 1770. There are two parts of the volume with distinct paginations and title-pages; one containing the papers on mathematics, and the other the miscellaneous papers. The editor's name is unknown. There is a set in University College, London.

19. *Miscellanea Mathematica: consisting of a large Collection of curious Mathematical Problems, and their solutions. Together with many other important Disquisitions in various Branches of the Mathematics. Being the Literary Correspondence of several eminent mathematicians.* § By C. Hutton. This periodical was issued in thirteen numbers, 1771-75, along with the parts of *The Diarian Miscellany* of which it has been termed a sixth volume (see under **1.1**). In

* Hutton, *Phil. and Math. Dict.* 1815; *D.N.B.* William Wales (1734 ?–1798), fellow of the Royal Society which sent him in 1769 to observe at Hudson Bay the transit of Venus, was the author of various works including a restoration of the determinate section of Apollonius (1772). See also *M.M.* vol. 60, 1854, pp. 436-437.

† Lawson, *Synopsis* (*l.c.*), pp. ii, 1, 3, 7, 9; *Miscellanea Mathematica* (No. 19), pp. 326, 328; *Math. Geom. and Phil. Delights* (No. 29), No. 2, p. 2; Wilkinson, *M.M.* vol. 61, 1854, pp. 244-245.

‡ Lawson, *Synopsis* (*l.c.*), a specimen of which was published in the *Oracle*; Wilkinson, *M.M.* vol. 50, 1849, pp. 561-565, and vol. 61, 1854, p. 124; *Cat. Per. Publs. Lib. Univ. Coll. London*, 1912.

§ P. J. Anderson, *Notes and Queries*, ser. 11, vol. 2, 1910, p. 347; Wilkinson, *M.M.* vol. 48, 1848, p. 83.

book form, with the date 1775, it contains 350 pages. (10·2 × 17·5 cm.) The title is sufficiently descriptive. Copies in B.M. and B.U.

20. *The Lady's and Gentleman's Diary, or, Royal Almanack*,* edited by Reuben Burrow † (1747-1792), and published by Thomas Carnan, who successfully contested in the courts the Company of Stationers' claim to a monopoly in such publications. Hence the publication is often referred to as *Carnan's Diary*, which appeared annually, under three different titles, for the years 1776-1788. For the last nine issues the title was simply *The Ladies Diary*. To avoid confusion the older work was then called the Old *Ladies Diary* to distinguish it from *Burrow's Diary* as Carnan's *Diary* was also called. An able and extensive contributor to this *Diary* was Jeremiah Ainsworth (1743-1784), grandfather of the novelist W. H. Ainsworth. ‡ There is a complete set of the *Diary* in American University, Washington, D.C., and the first seven issues are in the Library of the Royal Astronomical Society (*Catalogue*, 1886). At least three supplements to the *Diary* were published : *A Companion to the Ladies and Gentlemans Diary for the year* 1779, *A Companion to the Ladies Diary, for the year* 1780, and *A Companion to the Ladies Diary, for the year* 1781. These supplements may be seen in University College, London. Sets of the *Diary* and its *Companion* are excessively rare.

21. *The Lady's and Gentleman's Scientifical Repository : containing, Enigmas, Rebuses, Paradoxes, Philosophical, and other Useful Queries ; Arithmetical and Mathematical Questions and Problems, with their respective Solutions*,§ Newark, "By a Society of Mathematicians," of whom the chief editors are believed to have been William Spalton and Joseph Gales. (11·3 × 18·3 cm.) Number 1 (for January 1783) was published in December 1782, and number 10 for August-October 1784 was probably the last, the whole forming a volume of over 385 pages plus plates. There is a somewhat mutilated set in B.U. ; but a perfect set in the library of the Royal Society, London.

22. *Diaria Britannica ; or, the British Diary*,|| published annually (48 pp. each) for the years 1788-96, at Birmingham, 1787-95. (10·2 × 16·1 cm.) There are sets in B.U. and University College, London. Numbers 1-7 were edited by John Cotes and George Taylor ; and numbers 8-9 by Patrick Hall and John Cotes. On page 33 of number 1 is the following : "As the *British Miscellany*, printed in 1780 was discontinued, by reason of which several ingenious correspondents were disappointed, in not having their solutions inserted in the second number : we think it will be doing justice to those gentlemen in giving some of the *Questions* with their Solutions a place in our *British Diary*." This quotation acquaints us with yet another minor mathematical periodical.

23. *The Scientific Receptacle, containing curious original mathematical Questions and Solutions . . . selected from an extensive Correspondence, and intended*

* *Math. Geom. and Phil. Delights* (No. **28**), No. 2 (1792), p. 2 ; No. 4 (1794), p. 4 ; Wilkinson, *M.M.* vol. 51, 1849, pp. 244-247, 293-297, 350-357.

† Wilkinson, "The journals of the late Reuben Burrow," *Phil. Mag.* ser. 4, vol. 5, 1855, pp. 185-193, 514-522, vol. 6, pp. 196-204. Regarding these "journals" see A. De Morgan, *Notes and Queries*, ser. 3, vol. 5, 1864, pp. 107, 261, 361 ; Wilkinson's replies are on pp. 215 and 303. Memoir of Burrow by J. H. Swale, *M.M.* vol. 53, 1850, pp. 267-268. And finally references may be given to "Poggendorff," and to *D.N.B.* Burrow went to India in 1782, studied Sanskrit, and in his paper on "Hindoo knowledge of the binomial theorem" (1788) announced his intention very shortly to publish translations of the "Leelavotty" and "Beej Geneta." This intention was not carried out ; but in Strachey's translation of the latter (1813) there are various references to Burrow's work. Burrow's only separate work was a restoration of the Inclinations of Apollonius together with "the theory of Gunnery" (1779). See also under No. **16**.

‡ See Wilkinson, "Lancashire geometers . . ." (*l.c.*), and Brierley, "Lancashire mathematicians" (*l.c.*).

§ Wilkinson, *M.M.* vol. 53, 1850, pp. 412-415 ; *Cat. Per. Publs. Lib. Royal Society, London*, 1912 ; *Math. Geom. and Phil. Delights* (No. **29**), No. 2, p. (2).

|| Wilkinson, *M.M.* vol. 53, 1850, pp. 451-455 ; *Cat. Per. Publs. Lib. Univ. Coll., London*, 1912.

*to promote Emulation in the Ingenious of both Sexes,** Nos. 1-26, in 3 vols. 1791-1819, was edited by Thomas Whiting, "Schoolmaster, Keppel-House, near Brompton." (10·7 × 17·6 cm.) Complete copy at University College, London; volumes 1 (Nos. 1-9, 334 pp. + 8 plates)—2 (Nos. 10-18, p. 334 pp. + plates 9-17), Nos. 1-18, 1791-1809, in American University, Washington, D.C. No. 13 is dated March 25, 1797; No. 14, July 25; No. 15, March 20, 1798; No. 16, August; No. 17, ?; No. 18, July 15, 1809: No. 19, July 20, 1810. See also under **29**.

24. *The Stockton Bee; or, monthly miscellany . . . containing miscellaneous pieces, in prose and verse, queries, mathematical questions, etc.,*† Stockton, monthly, January 1793–December 1795. (9·6 × 15·7 cm.) The editor is unknown, but it seems probable that it was the printer and publisher J. Atkinson. Vol. 1 (414 pp. for 1793); vol. 2 (428 pp. for 1794); vol. 3 (431 pp. for 1795). Complete sets in B.M. and B.U.

25. *The Yorkshire Repository,*‡ York. A prospectus and twelve pages of preliminary matter were issued in October 1794. It ceased to exist with the first regular number (125 small quarto pages) about January 1, 1795. The editor is unknown, but it has been conjectured, with reason, that Thomas Leybourn had something to do with it. See under Nos. **1**. 4 and **26**.

26. *The Mathematical Repository,*§ edited by Thomas Leybourn ‖ (1770-1840) was published in two series, the first in fourteen numbers (10·4 × 17·2 cm.) 1795-1804, the second in twenty-four numbers (14 × 22·5 cm. untrimmed) 1804-35. In the first series numbers 2-8 were called *The Mathematical and Philosophical Repository*; for numbers 9-14 the title was *The Mathematical and Philosophical Repository and Review.* In the B.U. set of this series the original covers for all parts, except the first and the back cover of the seventh, have been preserved. The following dates occur on parts 3 to 14 respectively (3) March 1, 1797; (4) October 28; (5) March 26, 1798; (6) Sept. 1; (7) March 25, 1799; (8) Nov. 1; (9) June 1, 1800; (10) Nov. 18; (11) May 1, 1801; (12) Dec. 20; (13) Dec. 1, 1802; (14) May 1, 1804. It seems certain that No. (2) was published March 26, 1796; No. (1) appeared sometime in 1795. The first volume (426 pp. + 13 plates), dedicated to Hutton, is paged consecutively and appeared in Nos. 1-5, and hence was completed in 1798; a second edition has 1799 as the title-page. The second volume (472 pp. + 13 plates), dedicated to Maskelyne, was contained in Nos. 6-11. The third volume (268 pp. + 7 plates) in the remaining number is dated 1804. Each of these volumes is called *The Mathematical Repository.* Then there was a volume *A Review of Mathematical and Philosophical Books* (102 pp.) dated 1804 which appeared in Nos. 9-14. There were also two volumes of *The Philosophical Repository,* vol. 1 (376 pp. + 2 plates), 1801, in Nos. 2-11; and vol. 2 (128 pp.), 1804, in Nos. 12-14.

In the new series there were six volumes, each containing four numbers and each number containing three or four parts paged independently; these parts

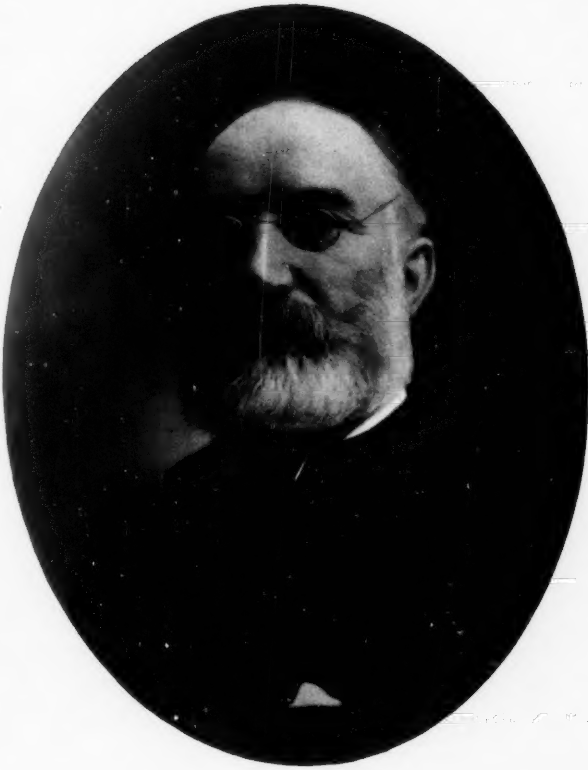
* T. Whiting, *Select Exercises*, 1803, advertisement at back; Wilkinson, *M.M.* vol. 53, 1850, pp. 363-367; *Cat. Per. Publs. Lib. Univ. Coll., London*, 1912.

† Wilkinson, *M.M.* vol. 52, 1850, pp. 63-65.

‡ Wilkinson, *M.M.* vol. 53, 1850, pp. 503-505.

§ Wilkinson, *M.M.* vol. 55, 1851, pp. 264-266, 306-310, 445-448; vol. 56, 1852, pp. 134-135, 145-147, 445-447; vol. 57, pp. 7-9, 64-66, 245-247, 291-294, 483. T. S. Davies, "Geometry and Geometers. No. VIII", *Phil. Mag.* ser. 4, vol. 2, pp. 445-446. In some places the curious error has been made of regarding this periodical of Leybourn as a continuation of *The Mathematical Repository*, vol. 1 (1748, 2nd ed. 1775), vol. 2 (1753), vol. 3 (1755), by James Dodson (*D.N.B.*), author of *The Anti-logarithmic Canon* (1742). Dodson's *Repository* was not a serial.

‖ Wilkinson has noted the following in *M.M.* vol. 67, 1857, p. 608: "'Samuel Thornoby,' =Thomas Leybourn, the signature being an anagram of his name. In number 4 of the *Mathematical Repository* (O.S.) the Editor (Mr. Leybourn) begs leave to inform Mr. S. Thornoby, that he believes Mr. Playfair's edition of the 'Elements of Euclid' the best for a learner. This was probably in return for the Professor's having given his consent for the republication of his 'Essay on the Origin of Porisms' in the first number of the *Mathematical Tracts*. . . . In some other parts of the Repository I find the Editor *awarding the prize to himself* under the disguise of this assumed name!"



E. M. Langley.

contain: (i) mathematical questions; (ii) original essays; (iii) mathematical memoirs extracted from works of eminence; (iv) Cambridge problems. The volumes were issued as follows: vol. 1, 1806, 38+256+56+78 pp.+10 plates; vol. 2, 1809, 22+258+80+34 pp.+12 plates; vol. 3, 1814, 14+206+88+46+38 pp.+7 plates; vol. 4, 1819, 14+168+136+76 pp.; vol. 5, 1830, 12+266+168+32+144 pp.; and vol. 6, 1835, 12+282+225+96+48 pp.+3 plates. There are complete sets of both series in B.U. and in University College, London (*Cat. Periods. Lib.*, 1912, with inaccurate note attached to the title).

This was a very notable publication. Davies gives us some interesting facts: "If, however, Mr. Leybourn lost money, he at the same time gained a high reputation from editing his Mathematical Repository. He thus obtained one great object of his early ambition which he could have gained in no other way; for the most devoted of his friends and admirers (amongst whom I place myself) will not contend for a moment that either his range of power or his mathematical acquirements could have gained for him that reputation in whatever other way exerted. The Repository (as well as the Diary) was *edited practically by his friends from its origin to its termination.* Dr. Hutton aided him in the outset of the first series, and subsequently Dr. Gregory and Mr. Lowry. In the earlier part of the new series he was dependent on the judgement of Messrs. Dalby,* Lowry,† Wallace,‡ and Ivory,§ with one or two others occasionally. In closing the fifth volume, and throughout the sixth, this office devolved partly on Mr. Woolhouse, but mainly on myself. During this latter period, too, the same may be said of the Gentleman's Diary. Dr. Gregory, however, supplied the almanac part of the Diary". See also under No. **2**. Latterly the *Repository* counted among its contributors many of the ablest English mathematicians of the time; for example, Adams, Babbage, Barlow, Davies, Gompertz, Gregory, Herschel, Horner, Hutton, Ivory, Lowry, Marrat, Nicholson, Peacock, Mrs. Somerville, and Wallace. There are sketches of all of these in *D.N.B.* Nearly a thousand problems were proposed and discussed, and they were connected with almost every field of pure and applied mathematics.

In the *Repository* for March 25, 1799, William Wallace gives as the first proposition of his "mathematical lucubrations" the following: "Let ABC be a triangle inscribed in a circle; from any point V in the circumference, let there be drawn perpendiculars VD, VE, VF to the sides of the triangle: the points D, F, E lie in a straight line". And thus we have the famous Wallace line of which the literature is now so extensive.

If the article by Davies on "Pascal's Conics" in the last volume of the *Repository* (1835) had been known to W. J. Macdonald, the article in the *Proc. Edinb. Math. So.* vol. 2, nearly fifty years later, would hardly have been offered for publication.

In volume 5 are various papers by Horner on his solution of numerical equations as partially developed in the *Philosophical Transactions*, 1819.

27. *The Student*,‖ Liverpool, four annual numbers (72 pp. each), 1797-1800. (10·2 × 16·6 cm.) Complete sets in B.U. and University College, London. John Knowles ¶ of Liverpool was editor of the first two numbers, and William Hilton,¶ of Saddleworth, of the latter two. John Butterworth and James

* Isaac Dalby (1744-1824), *D.N.B.* For a sketch mainly autobiographic, see *Mathematical Repository*, N.S. vol. 5, 1830, part 1, pp. 196-203. See also Wilkinson, *M.M.* vol. 61, 1854, pp. 243-244.

† John Lowry (1769-1850), *D.N.B.*

‡ William Wallace (1768-1843), *D.N.B.*

§ Sir James Ivory (1765-1842), *D.N.B.*

‖ Wilkinson: (i) *M.M.* vol. 48, 1848, pp. 83-84; (ii) *Educ. Times, Reprint*, vol. 13, 1870, p. 34; T. S. Davies, *Mathematician*, vol. 3, 1848, p. 83; *Cat. Per. Publs. Univ. Coll. London*, 1912.

¶ Wilkinson, "Lancashire Geometers" (*l.c.*), p. 128, and "Geometry in Lancashire" (*l.c.*), p. 57; Brierley, "Lancashire Mathematicians" (*l.c.*), p. 24, 27. Knowles wrote under the pseudonym "Non Sibi". Hilton died May 1826 (*Phil. Mag.* ser. 4, vol. 4, p. 204).

Wolfenden, two of Lancashire's very ablest mathematicians * were contributors to each of the annual numbers.

28. *The Gentleman's Mathematical Companion*,† annual for the years 1798-1827, published 1797-1826. (11 × 18·8 cm.) The first number of this periodical was issued in 1797 with the title *A Companion to the Gentleman's Diary, or a Preparative to that useful Work*. Charles Wildbore, editor of the *Gentleman's Diary*, disapproved of this and stated in the *Diary* for 1798 (p. 2) that he "wishes to discourage it all that lies in his power". Hence the next number of the *Companion* was changed in title, and to this the title of the first number was made to conform when the second edition was printed in 1809 (see "advertisment" in this edition, for which the diagrams were all redrawn and the text "carefully revised and corrected" by the editor, John Hampshire). Complete sets (with second edition of Part 1) in B.M. and B.U. Where may a copy of the first section of Part 1 be seen?

William Davis ‡ (1771-1807), the editor of the *Companion* from its commencement till his death, was a London bookseller. He published a number of important mathematical works, including a revised edition of Motte's translation of Newton's *Principia* (1803). His widow married J. S. Dickson, a bookseller and printer, who, in 1814, changed the name of his business to Davis & Dickson, who were not only the printers of the *Companion*, but also of Davis's edition of the *Gentleman's Diary* (No. **2**. 2). Davis was succeeded as editor, in the scientific department, by John Hampshire,§ author of papers and problems. The thirty issues of the *Companion* were also combined in five thick volumes with special title-pages, (1) 1798-1803; (2) 1804-9; (3) 1810-15; (4) 1816-20; (5) 1821-27. An appendix of twenty pages in (5) contains an index to the complete work. See also under **36**.

29. *Mathematical, Geometrical, and Philosophical Delights: containing Essays, Problems, Solutions, Theorems, etc. selected from an Extensive Correspondence*,‖ Parts 1-11, 1792-98 forming a volume of 32 + 228 pp. + 13 plates, with a special title-page dated 1797, but a preface dated Aug. 10, 1798. (15 × 19·2 cm.) The periodical is notable for numbering among its contributors many afterwards eminent. With a complete copy at B.U. is *The Poetical Delights* (36 pp.), which is not, apparently, one of a series. Both were edited by Thomas Whiting, editor also of *The Scientific Receptacle* (No. **23**).

30. *The Enquirer or Literary, Mathematical, and Philosophical Repository*,¶ London, numbers 1-11, 1811-13,** the last number being published in May. It was a quarterly publication (. . . × 17·5 cm.) of 750 copies, and numbers 1-8

* Cf. Brierley, "Lancashire Mathematicians" (*l.c.*), pp. 22, 28.

† Wilkinson, *M.M.* vol. 48, 1848, pp. 154-155, 224-226, 254-255, 279-281, 342-343, 401-402, 466-468.

‡ *D.N.B.*; *Gentleman's Mathematical Companion* for 1808.

§ *Gentleman's Math. Comp.* for 1826; Hampshire died April 1825, aged eighty-one years. He was recorded as a "man of great mathematical knowledge, integrity, worth, and unassuming parts".

‖ Wilkinson, *M.M.* vol. 48, 1848, p. 514; advertisement at the end of T. Whiting, *Select Exercises*, 1803.

¶ Wilkinson, *M.M.* vol. 49, 1849, pp. 5-6; B.M. and Library of Congress Catalogues.

** This date is correct, although one might get a different impression by reading the preface (dated "July 1st, 1815") to *The Leeds Correspondent*, vol. 1, 1815. One finds there a reference to: "the 'Enquirer', an excellent 'literary, mathematical, and philosophical repository', ably conducted by Mr. Marrat, and published quarterly at Boston, till *May last*, when it was discontinued" [the italics are mine]. This serial was never printed at Boston, but both of the editors lived there at the time of its publication; hence it is sometimes called *The Boston Enquirer*.

were edited jointly by William Marrat * (1772-1852) and Pishey Thompson † (1784-1862), number 9 by Marrat and J. Moore, and numbers 10-11 by Marrat alone. The eleven numbers formed three volumes. There are copies in Library of Congress and University College, London. See also under No. **32**.

31. *Quarterly Visitor,* ‡ Hull, Yorkshire, vol. 1 (292 pp.) containing six numbers, March 1813–June 1814; vol. 2 (283 pp.) containing six numbers, September 1814–December 1815 (last). (10·1 × 17·1 cm.) The editor was William Passman, a teacher of mathematics in Hull. The volumes contain memoirs, essays, and much other material not mathematical. The 135 questions belong mainly to the field of algebra and its applications. There are copies in B.M. and B.U.

32. *The Leeds Correspondent, A literary mathematical & philosophical Miscellany; consisting of original poetry and essays; curious anecdotes; . . . Mathematical Questions and answers, etc., etc. Communicated by Gentlemen of Scientific Attainments,* § Leeds, vol. 1 (four numbers, January 1814–July 1815, 14 + 288 pp.); vol. 2 (four numbers, January 1816–July 1817, 27 + 292 pp., title-page dated 1818); vol. 3 (four numbers, July 1819–July 1821, 8 + 288 pp. + 4 plates); vol. 4 (four numbers, January–October 1822, 12 + 328 pp.); vol. 5 (three numbers, January–July 1823, last, 4 + 252 pp., no title-page). (10·6 × 18·3 cm. untrimmed.) Copies in University College, London, and B.U. ‖ John Ryley ¶ (1747-1815) was the mathematical editor of the first volume; John Gawthorp, for the first three numbers of volume 2; and John Whitley ** (d. 1855) for the remaining numbers of the publication. The publisher edited the literary parts. It was an excellent periodical; it contained about 300 questions and their solutions. Since the first number of the *Correspondent* contained solutions of questions left over from the *Enquirer* (No. **30**) it may be regarded as continuing that serial.

33. *The Students' Companion, a Literary Mathematical and Philosophical Miscellany* †† was published at Blackburn, Lancashire, but only two numbers were issued, in October 1822 and April 1823. It has been referred to as *Blackburn Students' Companion* (*Lady's and Gentleman's Diary,* 1856, p. 70). It was edited by Charles Holt (see No. **37**) assisted by Henry Lightbown, a teacher of mathematics at Blackburn. I do not know of any library where this serial may be seen.

34. *The Liverpool Apollonius: or the Geometrical and Philosophical Reposi-*

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* *D.N.B.; Lady's and Gentleman's Diary,* 1853, p. 75; Marrat, mathematician and topographer, was for fifty years a contributor to such minor serials as the *Lady's Diary, Gentleman's Diary, Scientific Receptacle,* and *Leeds Correspondent.* He is the author of mathematical, topographical, historical, and astronomical books. He lived in New York, United States, where he was a teacher of mathematics 1817-1820, and editor of *The Scientific Journal,* of which nine numbers were published at Perth Arnboy, N.J. and New York, 1818-19; there is a set in the Boston Public Library (*Union List of Serials in Libs. of U.S. and Canada,* 1927).

† *D.N.B.*; Thompson the historian was a bank clerk at the time he was editor of *The Enquirer.* During 1819-46 he was a bookseller in the United States and an acquaintance of such men as Daniel Webster and Edward Everett.

‡ Wilkinson, *M.M.* vol. 48, 1848, p. 583.

§ Wilkinson, *M.M.* vol. 49, 1849, pp. 203-204, 303-306.

‖ B.U. has also *The Leeds Literary Observer,* of which there was a single volume (371 pp.) of nine numbers published in 1819. It was edited by the publisher, James Nicholls. This was a supplement to the non-mathematical parts of the *Correspondent.*

¶ J. Nicholls, *Leeds Correspondent,* vol. 2, pp. 97-103, 242-250; *D.N.B.; Reliquary,* vol. 11, 1871, p. 201.

** *Lady's and Gentleman's Diary,* 1856, p. 70; *Reliquary,* vol. 11, 1871, p. 201.

†† Wilkinson, *M.M.* vol. 49, 1849, pp. 437-438; also *Trans. Hist. So. Lancashire and Cheshir e* ser. 3, vol. 4, 1876, p. 87.

tory, by J. H. Swale.* There were only two numbers; the first published in November 1823 (116 pp. + plate of figs.) and dedicated to Thomas Leybourn "as a token of grateful recollection, of thirty years correspondence"; the second published in December 1824 (198 pp. + plate of figures) and dedicated to "Robert Adrain,† LL.D. (Professor of Mathematics and Natural Philosophy, Columbia College, New York)", "as a public expression of esteem for his worth and talents". (14 × 22 cm.) Both volumes are in B.U. Neither, so far as I know, is in an English public library; the first one is excessively rare, but the second volume can be seen in Mr. Greenstreet's collection at Burghfield. Their contents are indicated by Wilkinson, and they are especially valuable because of Swale's original and elegant geometrical contributions.

35. *The Scientific Receptacle, a literary, mathematical, and philosophical repository, containing original essays and poetry* ‡ was a quarterly, completed in four numbers (6 + 266 pp. Jan.–Oct. 1825), and published at Holbeach, Lincolnshire. It was edited by Henry Clay, teacher of mathematics at Moulton, and was intended "to elicit the latent spark of genius, to create a generous and laudable emulation among the youthful votaries of science, to disseminate useful and entertaining knowledge, and to open a field for the recreation and exertion of the adept in mathematics". There is a copy in University College, London (*Cat. Per. Publs. Lib.* 1912).

36. *Enigmatical Entertainer and Mathematical Associate*,§ number 1, for the year 1828, appeared in November 1827 to replace *The Gentleman's Mathematical Companion* (No. **28**), which ceased publication in the previous year. Four annual numbers appeared. The first number and the *Enigmatic Entertainer* parts of the other three are paged continuously (1-166); then the *Mathematical Associate* parts of numbers 2-4 are also paged continuously (1-88, which includes an index to the whole). (10·3 × 17·1 cm.) The work was edited by Paul Ninnis. Mathematically it is much less interesting than the *Companion*. There are copies in B.M. and B.U., the latter being Wilkinson's copy.

37. *The Scientific Mirror, a Literary, Mathematical, and Philosophical Repository*,‖ was started at Bolton, Lancashire, and only two numbers were published; in Nov. 1829 (8 + 22 + 6 pp.) and June 1830 (4 + 30 + 15 pp. + 1 plate). (12 × 17·1 cm.). It was edited by Charles Holt, teacher of mathematics at Houghton, near Blackburn; see also No. **33**. There are copies in American University, Washington, D.C., and in University College, London (*Cat. Per. Publs. Lib.* 1912). It was "designed as an introduction to the Mathematics in general; to lead the ingenious youth to an acquaintance with the sciences, and the application of them to the various purposes of Society".

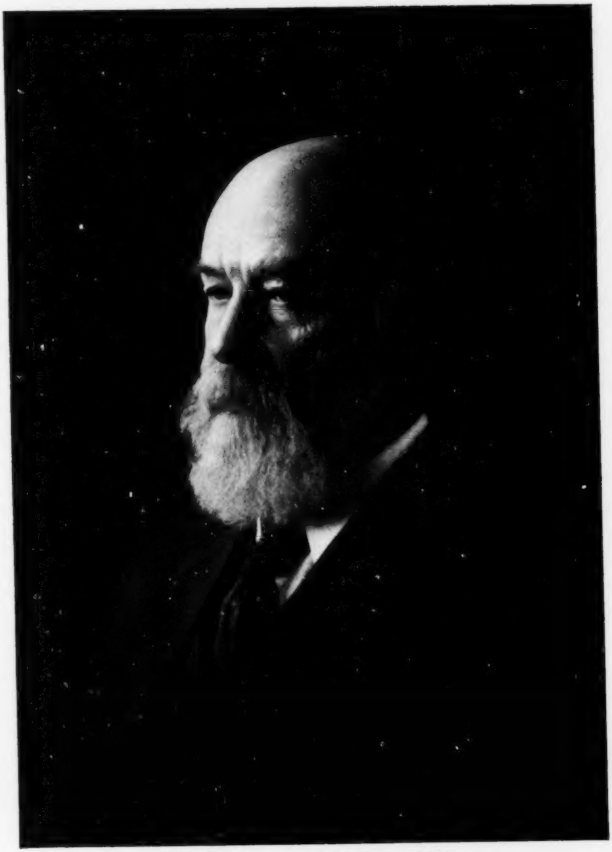
* T. S. Davies, *The Mathematician*, vol. 3, 1849, pp. 317-318; T. S. Davies, *Phil. Mag.* ser. 4, vol. 1, 1851, pp. 536-544; Wilkinson: (i) *Phil. Mag.* ser. 4, vol. 4, 1852, pp. 29-33, 201-209; (ii) *M.M.* vol. 58, 1853, 306-307, 327-328; (iii) *Lancashire and Cheshire Hist. So. Trans.* vol. 7, 1855, pp. 143-164; vol. 10, 1858, pp. 169-182 + 1 plate; C. W. Sutton and J. H. K. *Notes and Queries*, ser. 11, vol. 1, 1910, pp. 192-193. Swale was a schoolmaster in Liverpool from 1810 until his death in 1837 at the age of sixty-two. In England he was the most independent and original geometer of his time. His methods were Euclidean. Apart from many contributions to periodicals he published *Geometrical Amusements: or a course of Lessons in Construction and Analysis in three parts. Part 1*, London, 1821, 256 pp. Davies characterises this work as, "undoubtedly, one of the most original and remarkable books on geometry that has appeared since the time of Stewart and Simson". Only part 1 was published.

† J. L. Coolidge, *Amer. Math. Mo.* vol. 33, 1926, pp. 61-76. The first part of Adrain's article, "View of Diophantine algebra", in *Mathematical Correspondent*, Reading, Pa. 1806, seems to have been reprinted in the second volume of *The Liverpool Apollonius*.

‡ Wilkinson, *M.M.* vol. 49, 1849, pp. 367-368.

§ "Prospectus" (4 pp.) dated November 9, 1826, and issued with *The Gentleman's Mathematical Companion* for 1827; Wilkinson, *M.M.* vol. 54, 1851, pp. 449-450, 474-475, 492-494; *Ladies' Diary*, 1833, p. 37. It is usually referred to as *Mathematical Associate*.

‖ Wilkinson, *M.M.* vol. 49, 1849, pp. 523-524. It has been referred to as *The Bolton Scientific Mirror* (*Lady's and Gentleman's Diary*, 1856, p. 70).



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PROF. A. LODGE.

38. *The Private Tutor and Cambridge Mathematical Repository; comprising illustrations and examples in every branch of the Mathematics; with Essays, Problems, Solutions, and other contributions . . .*, Cambridge, vol. 1 (1830, thirteen numbers, 478 pp.); vol. 2 (1831, seven numbers, 418 pp.). (13·8 × 22 cm.) The editor J. M. F. Wright * was the author of *A Commentary on Newton's Principia . . . Designed for students at the Universities*, 2 vols. (1828), *Solutions of the Cambridge Problems, from* 1800 to 1820, 2 vols. (1825; 2nd. edn. 1836), and other volumes. The first number opens as follows: "The design of this Periodical is to supply Mathematical Students with the usual aids of a Private Tutor; that is, to afford them numerous illustrations and examples of the several branches of abstract and physical science; to give occasionally superior proofs of fundamental propositions, and brief supplementary essays on those subjects which are imperfectly treated in the works adopted in Lecture-rooms". And later: "With respect to the present performance, its design was merely to form a Supplement to Wood's excellent and elegant treatise on Algebra"; this fact is brought out on the title-page of vol. 2. There are copies of this periodical in B.M. and B.U.

39. *The York Courant* † was a newspaper which, from 1829, included philosophical queries and mathematical questions. The column was edited by Thomas Turner Tate ‡ (1807-88) till 1840, when he left York to become master of the mathematical and scientific department of Battersea Training College. His successor as editor of the *Courant* was William Tomlinson, then mathematics teacher of Archbishop Holgate's Grammar School, and afterwards of St. Peter's College, York. The column was discontinued in 1846.

40. *The Northumbrian Mirror; or, Young Student's Literary Mathematical Companion, forming an introduction to the Ladies Diary*,§ vol. 1 (358 pp.), Nos. 1-6, Jan. 1837–May 1838, title-page dated 1837, index; vol. 2 (420 ? pp.), Nos. 7-13, Sept. 1838–Sept. 1840, title-page dated 1838; both of these volumes (10·3 × 17 cm.) were printed at Alnwick. The remaining two numbers (Jan., July, 1841) were printed at Newcastle-upon-Tyne. B.M. and Cornell University each have vols. 1-2. I know of no public library which has the numbers of the new series. "The motives that led to the publication of the 'Northumbrian Mirror' were simply, the great and acknowledged want of a medium, through which, the younger and less experienced Mathematician might aspire after scientific renown; and to supply him, at regular intervals, with fresh and agreeably diversified subjects both of practical utility and of theoretical interest, so as to animate him in his course, stimulate his exertions, call out his latent energies, keep his intellectual weapons from the rust of inactivity, and lead him forward in an agreeable path to the more highly cultivated regions, and the loftier eminences of Scientific thought." The first thirteen numbers were edited by Rev. W. Telfer of Alnwick, and the last two by Stephen Fenwick,‖ for a time mathematical master at the Royal Military Academy, Woolwich.

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* W. W. R. Ball., *Cambridge Papers*, London, 1918. Wright was the author of a pseudonymous autobiographical work, *Alma Mater: or Seven Years at the University of Cambridge*, 2 vols. (1827), which describes in detail the mathematical teaching of the time in the University.

† Wilkinson, autobiographical sketch, *Trans. Hist. So. Lancashire and Cheshire*, ser. 3, vol. 4, 1876, p. 86; Brierley, *Papers Manchester Lit. Club*, vol. 4, 1878, pp. 20-30; *L. and G. Diary*, 1853, p. 70. In the B.M. *Catalogue* it seems to be suggested that *The York Courant* was amalgamated with *The York Herald* in 1827 and continued under the latter title for the next sixteen years, at least, that is, during the whole period 1829-46. At present I have no explanation to offer.

‡ *D.N.B.*; Tate was the inventor of the double piston air pump that is known by his name, and his *Principles of Geometry, Mensuration, Trigonometry, Land Surveying, and Levelling* (1848) was translated into Hindustani. See *Lady's and Gentleman's Diary* for 1848, p. 70. His mother's name was Turner. Was it her father, Thomas Turner, who was editor of *Miscellanea Curiosa* published at York (see No. 5)?

§ *Ladies' Diary* for 1839, p. 20; *Lady's and Gentleman's Diary* for 1842, p. 66; Wilkinson, *M.M.* vol. 59, 1852, pp. 506-507, 528-530.

‖ Fenwick was the author of *Mechanics of Construction* (1861), and joint-author with Rutherford of *Elementary Course of Math. prepared for the Royal Military Academy* (1850).

41. *The Mathematician* was started to meet the need of a periodical devoted wholly to mathematics of not too advanced a nature.* Had Leybourn's *Repository* (No. **26**) been continued, the necessity for *The Mathematician* would not have existed. Nevertheless the new journal displayed differences. The department of mathematical questions was curtailed; the only questions of this kind which were published were "such as involve some new principle, or require for their solution some new modes of investigation,—such as either lead to results remarkable for their unexpected simplicity, elegance, and symmetry; or tend to the extension of an old, or to the commencement of a new, and valuable course of inquiry". The editors planned also (1) "to simplify both the investigations and modes of actual operation of various elementary processes, and to furnish good models for the young student's initiation in conducting his own researches"; and (2) "to develope in sufficient detail those *methods in geometry*, which have been devised and carried out to considerable extent by the Continental Mathematicians". Three volumes of this excellent periodical were published: vol. 1 (346 pp.), six numbers, Nov. 1843–July 1845, title-page dated 1856; vol. 2 (348 pp.), six numbers, Nov. 1845–July 1847, title-page dated 1856; vol. 3 (398 pp.), seven numbers, Nov. 1847–Sept. 1850, title-page dated 1856. (15 × 18·8 cm.) T. S. Davies † (1795-1851) was an editor of the first volume only; William Rutherford ‡ (1798 ?-1871) and Stephen Fenwick (see No. **40**) were editors of all three. They recognised before starting on their undertaking that financial loss was likely to accrue, but they were backed by personal friends who formed themselves into a society for raising a small annual fund for meeting deficits.

42. *The Educational Times*, in so far as it interests us, was published under this title, vols. 1-69, 1847-1915. Consecutively numbered mathematical questions seem to have begun with the issue for August 1849 (vol. 2, p. 255), although questions, solutions, and mathematical papers appeared occasionally before this. Wilkinson has stated,§ "When the *Educational Times* was established, I was one of its earliest promoters and had the principal share in introducing the mathematical departments into that journal. This portion was first edited by Richard Wilson,‖ D.D., author of an excellent treatise on trigonometry [Cambridge, 1831]". As far back as 1858, at least, it would seem as if this editorship had passed into the hands of Wm. John Clarke Miller ¶ (1832-1903), who, in 1861, conceived the idea of devising some plan

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* England's first exclusively mathematical periodical of a research character, *The Cambridge Mathematical Journal*, was started just six years before, and was destined to continue under changed names to the present time.

† *D.N.B.* Apart from references there given are the following: *Lady's and G. Diary* for 1852, p. 70; [Wilkinson] *Westminster . . . Review*, vol. 55, 1851, Amer. ed. pp. 35-42; *M.M.* vol. 62, 1855, pp. 271-275; vol. 63, 1855, pp. 220-223; vol. 64, 1856, pp. 27-28; "Poggendorff", vol. 1, 1863; De Morgan, *Budget of Paradoxes*, 1st ed. 1872, pp. 350, 351; 2nd ed. vol. 2, 1915, pp. 151-152. Davies was one of the mathematical masters at the Royal Military Academy, Woolwich, from 1734 till his death after six years of illness. He was the author of many problems and papers in minor serials, as well as papers in such publications as *Trans. R. So. Edinb.*, *Phil. Trans.*, and *Cambridge and Dublin Math. Jl.* He wrote under many pseudonyms; some of them are: Pen-and-Ink, Shadow, Centurion, Miss L. L., Dunelmensis Bathonensis, Zephyr, Rev. Peter Twaddleton, Figaro, Sidrophel, Diedrich Knickerbocker, S.S.S., and Brown Rappee (*M.M.* vol. 67, 1857, p. 608).

‡ *D.N.B.*; Rutherford taught at the Royal Military Academy, Woolwich, 1838-64, and wrote a number of mathematical works.

§ Autobiographical sketch in *Trans. Hist. S. Lancashire and Cheshire*, ser. 3, vol. 4, 1876, pp. 86-87. There is a reference, p. 89, to Miller being the editor of the mathematical part of *The Key*, another minor mathematical serial which lasted for two years at least.

‖ Author of a dozen volumes and pamphlets, theological and literary, 1830-40, 1870, 1873 (B.M. *Catalogue*).

¶ B. F. Finkel, *Amer. Math. Mo.* vol. 3, 1896, pp. 159-163 + portrait of Miller; Miller, preface to 2nd ed. of *Math. Q. and Sols.* vol. 1, 1886; *Bibliotheca Mathematica*, ser. 3, vol. 4, 1903, p. 111; B.M. *Catalogue*. Miller was the author of *Essays and Nature Studies with Lectures*, edited with an introd. by H. K. Swann, London, 1899. For a time Miller was a professor at Huddersfield College, Yorkshire, but in 1876 he became registrar and secretary of the General Medical Council. In 1887 he founded the Richmond Athenaeum, of which he was long president.

whereby the contributions to the mathematical columns might be preserved, apart from other matter, in a more convenient form. After ascertaining the views and desires of contributors and obtaining the necessary promises of support, there was published in July 1864 the first volume of *Mathematical Questions with their solutions. From the "Educational Times"* (92 pp. $15{\cdot}2 \times 24{\cdot}5$ cm.), the contents being mainly a reprint of what had appeared previously,* July 1863 to June 1864. Subsequent volumes appeared at half-yearly intervals to the end of 1918. The second volume had added in its title, *With many Additional Solutions not published in the Educational Times*, which was changed to *With many Papers and Solutions* . . . in vol. 3; the equivalent of this persisted to the end of 1915. The 110 volumes of this very extraordinary publication were issued in three series: ser. 1, vols. 1-75, 1864-1901, each about 110-220 pp.; ser. 2, vols. 1-29, 1902-15 ($15{\cdot}1 \times 24{\cdot}3$ cm.); ser. 3, vols. 1-6, 1916-18. The volumes of the third series (the same format as the second) were no longer reprints of what had appeared in columns of the *Educational Times*, but were reprints of a monthly periodical entitled *Mathematical Questions and Solutions in Continuation of the Mathematical Columns of "The Educational Times"*; only thirty-six monthly parts were issued. W. J. C. Miller was the sole editor of the first sixty-six volumes and joint-editor of vol. 67 with "D. Biddle, M.R.C.S., fellow of the Royal Statistical Society," who was sole editor of vols. 68-75. Miss Constance I. Marks, B.A., was the editor of the thirty-five volumes in the second and third series. In 1886 Miller wrote as follows:

"In the volumes there have appeared, from time to time, articles in almost all branches of Mathematics; and the leading Mathematicians of this and other countries have, by their contributions or otherwise, helped forward the work with their encouragement or their support.

"Of those who aided the Editor in the production of his first volume, few are now left, but to these few especial acknowledgements are due. Mr. Tucker has been a valued correspondent down to the present time; and Dr. Hirst † after enabling us by his articles to develope those elegant branches of Geometry in which our readers take so deep an interest, collected and published his contributions in a separate volume.‡

"The second article in this volume is from the pen of Professor Cayley, to whom we owe other important articles in this and following volumes; and the comparatively new theory of Local Probability has been largely discussed by Mr. Woolhouse, Colonel Clarke and Professor Crofton.

"To Professor Sylvester the Editor would desire to express his deepest thanks for never-failing aid and encouragement. From the very earliest sheets of this volume in 1863 on to those now passing through the press, there have been but three or four numbers of the Journal—and these through the merest inadvertence—without at least one article of his being among the number. §

"To the elegance and originality of the contributions of one prematurely lost to Science, these volumes bear ample testimony. The very earliest Mathematical Solutions by the late Professor Clifford appeared in this volume, and his articles went on increasing in number and value through many a succeeding volume."

Over eighteen thousand questions were proposed for solution in the above-

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* The page of the *Educ. Times* contained three columns previous to July 1864, when wider columns, two to the page, were published. The first volume of the *Reprint* had two columns to the page; but later volumes had but one somewhat different in format ($13{\cdot}6 \times 21{\cdot}2$ cm.). To this style the second edition of volume 1 (1886, 164 pp., with "some needed corrections and improvements") was made to conform.

† Thomas Archer Hirst (1830-92). "Poggendorff"; *D.N.B.*

‡ *Geometrical Contributions to the Educational Times*, 63 pp. 1875 (Hodgson).

§ It is to be hoped that some one will bring out a volume containing all of Sylvester's contributions to the *Educational Times*. These go back to 1853 at least. None of them, except such as may also have appeared elsewhere, are included in his *Collected Mathem. Papers*.

mentioned volumes, and solutions for most of them were published. Although 1400 questions had been proposed before July 1863, the solutions of many of them appeared in later volumes. B.U. is the fortunate possessor of large volumes, formerly the property of the editors of the *Reprint*, containing practically all of the questions from 482 to 18,769, with references to the *Reprint* volumes wh re the solutions had been published. A volume indexing this material would be of immense service. But all that has appeared in print so far has been a subject-index in each of the volumes, ser. 2, vols. 6-28; ser. 3, vols. 1-4.

43. *Association for the Improvement of Geometrical Teaching, Reports,** 1-19 (last), 1871-93, 1-11 being printed at Birmingham, and 12-19 at Bedford. ($13{\cdot}6 \times 21{\cdot}4$ cm.) Assuming that the "honorary secretaries" were the editors of the *Reports*, we have as an editor: R. Levett,† 1-9; E. F. MacCarthy, 1-5; R. Tucker,‡ 6-8; E. B. Sargant, 9-11; J. B. Lock, 10; E. M. Langley,§ 11-19; C. Pendlebury,‖ 12-19. The *Reports* (20 to 80 pp. each) are the proceedings of the A.I.G.T. for the period in question, and contain a number of interesting brief notes and papers.

In order to provide a more adequate medium for the publication of papers the A.I.G.T. (which in 1881 had widened its basis so as to include all branches of elementary mathematics) started the publication of *The Mathematical Gazette. A terminal Journal for Students and Teachers* ($20{\cdot}9 \times 26{\cdot}7$ cm.). Of this there were six numbers (60 pp. + plates), April 1894–Oct. 1895. This was followed by *The Mathematical Gazette* ($14 \times 22{\cdot}5$ cm.), vol. 1 of which embraces Nos. 7-24, Apr. 1896–Dec. 1900 (434 pp.; index, Nos. 1-24). In March 1897 the name of the A.I.G.T. was changed to the Mathematical Association, which still exists, with the *Gazette* as its official organ. The current volume, 14, began in January 1928; the four previous volumes covered biennial periods.

The editors of the *Gazette* have been as follows: E. M. Langley, April 1894–Oct. 1895; F. S. Macaulay,‖ Apr. 1896–Dec. 1898, and then as associate editor to the present time; E. T. Whittaker,‖ associate editor Feb. 1900 to the present; F. W. Hill, associate editor Oct. 1897–Oct. 1898; H. W. Lloyd Tanner,¶ associate editor Feb. 1899–May 1913; W. E. Hartley,** associate editor, co-operating in editing problems and solutions, Dec. 1900–May 1906; W. J. Greenstreet,†† associate editor Oct. 1897–Oct. 1898, editor Feb. 1899 to the present time. In the course of a year the *Gazette* presents a very wide range of material of much interest to all mathematicians. Among minor mathematical serials throughout the world at the present time, Schotten and Lietzmann's *Zeitschrift für mathematischen und naturwissenschaftlichen Unterricht* and Greenstreet's *Mathematical Gazette* are in a class by themselves by virtue of the care with which they are edited. But of these two the palm must certainly be awarded to the *Gazette* because of the fine feeling and literary flavour so much in evidence, especially during the latter part of its editor's thirty years of service.

"Gesundheit dem bewährten Mann,
Dass er noch lange helfen kann!"

Brown University, April 6, 1929. R. C. ARCHIBALD.

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* E. M. Langley, "The early history of The Mathematical Gazette," *Math. Gazette*, vol. 7, 1913, pp. 134-136.

† C. H. P. Mayo and C. Godfrey, *Math. Gazette*, vol. 11, 1923, pp. 325-329 + portrait.

‡ Robert Tucker (1832-1905). G. B. Halsted, *Amer. Math. Mo.*, vol. 7, 1900, pp. 237-329, portrait opp. p. 277. M. J. M. Hill, *Proc. London Math. So.* ser. 2, vol. 3, 1905, pp. xii-xx.

§ *Math. Gazette*, Jan. 1913, vol. 7, in honour of Langley; biographical sketch and portrait.

‖ *Who's Who, 1929.*

¶ *Who Was Who, 1897-1916*, H. W. L. Tanner (1851-1915).

** *Math. Gazette*, vol. 9, 1917, p. 144, "H. M. S. Vanguard, July 9, 1917."

†† Biographical notes and portrait, *Math. Gazette*, vol. 7, 1913, pp. 28-29; *Who's Who, 1929.*

The Rev. J. J. Milne.

SUPPLEMENTARY NOTE, MAY 3, 1929.

Other minor serials are:

S 1. *Newcastle General Magazine*, Newcastle-upon-Tyne, 1747-1757.

S 2. *The Gentleman's Magazine*, 1751-1768 at least.

S 3. *Court Magazine*, Sept. 1761-Nov. 1765.

S 4. *London Magazine*, Oct. 1774-June 1785 (last).

S 5. *The Mathematical and Philosophical Repository: consisting of Essays, Theorems, Problems, Solutions,... selected from an extensive correspondence* by John Davison, 1789-91.

S 6. *Newcastle Magazine*, Newcastle-upon-Tyne, Sept. 1820-March 1831.

S 7. *Western Miscellany, a Journal of Literature, Science, Antiquities, and Art for the West of England*, Nos. 1-13 (last), Exeter, 1849-1850.

S 8. *The Alnwick Journal*, ed. by J. A. H. Tate, Alnwick, 1862-1863.

INDEX TO SERIALS.

(*The numbers refer to the numbers of the serials.*)

INDEX TO EDITORS.

(The numbers refer to the numbers of the serials.)

WHAT IS ENERGY?

By Prof. E. T. Whittaker, F.R.S.*

A well-known treatise on Physics written in 1892 begins with the words: "A careful study of phenomena reveals to us only two things, or entities, as having an actual and objective existence. These two things are Matter and Energy. So far as we can determine, therefore, these two entities, in their various forms, make up the whole of the physical universe."

Such was the accepted doctrine in the latter part of the nineteenth century. I want to show how it originated and to what extent we have now departed from it.

The history begins in this way. Soon after Newton's discovery of the laws of motion, attention was drawn to some interesting properties possessed by the quantity which is obtained when we multiply the mass of a moving particle into the square of its velocity; and Leibniz in 1695, thinking that this quantity deserved a name of its own, called it the *vis viva.* Not long afterwards, John Bernoulli arrived at the idea of a *conservatio virium vivarum,* by which he meant that when the *vis viva* disappeared (*e.g.* when a pendulum was momentarily at rest at the extremity of its swing) the *facultas agendi* or capacity for doing work was not annihilated, but existed in some other form (as we should now say, in the form of potential energy). This was perhaps the earliest explicit statement of the doctrine that the "capacity for doing work" is a definite quantity which can exist in different but interchangeable forms.

During the eighteenth century the science of theoretical dynamics was tremendously developed, and assumed what we now call its classical form: in Lagrange's *Mécanique analytique* of 1788 we find the equations of motion of a system expressed in terms of the *semi-vis-viva* T and the potential-function V in the form

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_r}\right)-\frac{\partial T}{\partial q_r}=-\frac{\partial V}{\partial q_r},$$

with *le principe de la conservation des forces vives*

$$T+V=\text{Constant}.$$

Lagrange did not say that T and V are to be regarded as different manifestations of the same physical entity, but this idea is explicit in Carnot's *Principes de l'Equilibre et du Mouvement* of 1803, where V is spoken of as the *force vive latente.*

The dynamical principle of the conservation of *vis viva* was extended to another branch of physics by Fresnel in his celebrated memoir on the reflexion and refraction of light presented to the French Academy in 1823. Fresnel supposed light to consist in the vibrations of an "aether," so that the idea of *vis viva* could be associated with it: as the nature of the aether was unknown, he was compelled, in order to get a sufficient number of equations to solve the problem, to assume that the vibrations satisfied certain simple relations such as were already familiar in ordinary dynamics: and in particular, he assumed that the principle of conservation of *vis viva* was satisfied, *i.e.* that the amount of *vis viva* brought in one second to the refracting interface by the incident ray was equal to the amount of *vis viva* carried away from the interface in one second by the reflected and refracted rays together. Here for the first time we meet with the idea of *vis viva* "streaming" like a moving liquid.

It was, however, the general adoption of correct views as to the nature of heat which did more than anything else to elevate *vis viva,* or *energy* as it had been named by Thomas Young in 1807, into a concept dominant throughout the whole of nature. Joule at Manchester in 1840-49 showed that the

* From a lecture to the Edinburgh University Physical Society, 16th January, 1929.

quantity of heat capable of raising the temperature of a pound of water by 1° Fahr. was equivalent to the energy represented by the fall of 772 pounds through a space of one foot. Thus an equivalence was set up between dynamical *vis viva*, dynamical "latent energy" of position, and heat; chemical and electrical energy were also brought into the scheme, and in Helmholtz's tract *Ueber die Erhaltung der Kraft* of 1847 the conservation theorem was asserted as a universal principle, namely, that the whole amount of energy in the universe, or in any limited system which does not receive energy from without, or part with it to external matter, is invariable.

We have now arrived at an epoch when it became customary to think of energy as invested with all the attributes of "thinghood": and we have to consider what these attributes are.

We may, I think, say that the qualities which we associate with physical reality are:

1. *Location in space:* anything which has physical reality must be somewhere.
2. *Conservation:* it must exist in a certain amount, and the amount at any place can only be changed by bodily transference.

Mathematically these conditions may be put in the following form: that it must be possible to assign a number ρ which is taken to represent the density of the "thing" in question, *i.e.* the amount of it per unit volume at the time t at the place (x, y, z), and also to assign a vector (u_x, u_y, u_z) which represents the "flux" or current of the thing: and the four numbers (ρ, u_x, u_y, u_z) must satisfy the equation of conservation:

$$\frac{\partial \rho}{\partial t}+\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}+\frac{\partial u_z}{\partial z}=0, \qquad (1)$$

which is like the "equation of continuity" in hydrodynamics.

Now the left-hand side of equation (1) belongs to the type of mathematical expressions called "divergences"; and the important point to notice is that when the physicist says that so-and-such-a-thing has "physical reality," this is merely a picturesque way of stating a mathematical equation, namely, that the divergence of the four numbers (ρ, u_x, u_y, u_z) is null. Whenever we come across a divergence-relation of the type (1), we can, if we please, use it to endow a mathematical expression with "physical reality."

The question of the localisation of energy has always been specially prominent in connexion with electric and magnetic phenomena. Granted that energy, of a certain known amount, exists in connexion with electric and magnetic systems, where is it localised? In the charges and magnets, or outside them? The first great discovery in this domain was that of William Thomson (Lord Kelvin) in 1853, that the energy of a magnetic system can be accounted for by supposing an amount $\mu H^2/8\pi$ of energy to be stored in each unit volume of the medium, where H denotes the magnetic force and μ the permeability. A similar expression was found for the energy of an electric system; and the localisation was harmonised with all known facts by the theorem, found independently by Poynting and Heaviside in 1884, that the flux of electric and magnetic energy at any place is represented by the vector-product of the electric and magnetic forces, divided by 4π. When optical phenomena are interpreted as being really electromagnetic, Poynting's result agrees with Fresnel's formulae of 1823 for the streaming of energy along beams of light.

Considering this system of energy-storage and energy-flux in the "aether," J. J. Thomson pointed out in 1893 that in order to make the whole representation consistent from a mechanical point of view, it was necessary to suppose that the aether was also a storehouse of momentum, obeying the same laws as ordinary mechanical momentum and transformable into it: the

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W. J. Greenstreet.

amount of momentum per unit volume he found to be, in free aether, proportional to the vector-product of the electric and magnetic forces, and therefore exactly proportional to the Poynting flux. Now this is just what we have in an ordinary fluid, where, of course, the flux of energy and the momentum are proportional to each other; and thus it was realised that the mysterious energy of the electromagnetic field was much more like the ordinary *vis viva* of dynamics than anyone had previously supposed.

The idea of a close connexion between electrical energy and the *vis viva* of material bodies had already been suggested in 1881 by J. J. Thomson as an inference from his investigation of the motion of an electrically-charged sphere. If a sphere of radius a has a charge e uniformly spread over it, its electrostatic potential energy, which we may denote by U, is equal to $\frac{e^2}{2a}$; and if it is moving in a straight line with uniform velocity v, the moving charge creates a magnetic field whose magnetic energy is $\frac{e^2v^2}{3ac^2}$ or $\frac{2U}{3c^2}v^2$, (where c denotes the velocity of light), so regarding this as kinetic energy since it is proportional to the square of the velocity of the moving body, the sphere when charged behaves as if it had an additional mass of amount $\frac{4U}{3c^2}$. This result of Thomson's was however obtained by using a conception of a rigid body which was found later to need modification in the light of the theory of relativity; when this defect in the calculation is remedied, the additional or "electromagnetic" mass is found to be of amount $\frac{U}{c^2}$. It was seen from this example that electrostatic potential energy behaved as if it had the property of inertia, the equivalent mass being $\frac{1}{c^2}\times$ the energy. Poincaré in 1900 adumbrated the principle that *all* energy has the property of inertia, and this was further supported in 1904 by Hasenöhrl, who proved that a box with perfectly reflecting walls, when filled with radiant energy, behaves as if it had additional mass. The principle was more definitely enunciated and justified by Einstein in 1905 and Planck in 1908, and we may now take it as an established fact that $(3\cdot10^{10})^2$ *ergs of energy have a mass of one gram.*

About this time (1904-1908) it was found that the relation

$$\text{Energy} = c^2 \times \text{Mass}$$

holds even when the mass is that of an ordinary particle: for as a consequence of the Principle of Relativity, if a particle has velocity v, its momentum is

$$\frac{mv}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}, \qquad (2)$$

where m is a constant characteristic of the particle and independent of its velocity (this value of the momentum is, of course, approximately mv when the velocity is small compared with the velocity of light): and the "mass," defined as the quotient of the momentum by the velocity, is therefore

$$\frac{m}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}},$$

a formula which has been verified experimentally by the work of Kaufmann, Bucherer, and others, on the deflexion of the moving electrons of β rays. The kinetic energy is found to be

$$\frac{mc^2}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}} \qquad (3)$$

(which, when the velocity is small compared with the velocity of light, is approximately a constant $+\frac{1}{2}mv^2$), and therefore we have in this case also

$$\text{Kinetic energy} = c^2 \times \text{Mass}.$$

Planck and Einstein now went on to review all Physics in the light of the principle that energy possesses inertia; and in 1907 put forward the suggestion that since ordinary masses, which have inertial properties, have also gravitational properties, it was therefore likely that energy, of any kind, would give rise to a gravitational field and would be subject to attractive forces from other gravitating bodies. Since light is a form of energy, they inferred that a ray of light, in passing near the sun, would be deflected by the solar gravitational field. Such was the origin of the discovery which excited so much popular interest when it was verified observationally by the eclipse expeditions of 1919 and 1922.

It may be remarked in passing that since 1907 the point of view with regard to this phenomenon has somewhat changed. A modern physicist, if asked to explain why light-rays bend when they pass close to the sun, would reply that it is because space is curved or distorted by the sun's gravitation, and the light, like everything else, is affected by the distortion of the space through which it is travelling; just as all the traffic that passes along a curved road moves in the same curve, without regard to the particular kind of traffic it may be; thus the emphasis nowadays is on the distortion of space and not at all on the qualities of the light itself. But in 1907 the distortion of space was as yet undreamt of, and the reason given for the deflection of light was simply that light was energy, that energy had inertial properties, and that whatever had inertial properties had also gravitational properties.

Let us now examine more closely, in the light of the modern doctrine that gravitation is essentially a distortion of space, why it is that energy—even potential energy—is subject to gravitational attraction. The matter can be simply elucidated in the case of a system of two point-charges of electricity. Let e and e' be two electric charges at P and P' respectively, and denote the length PP' by r. According to the old electrostatics, each of these charges will exert a ponderomotive force on the other, the forces being equal and opposite in the line PP', and being of amount $\dfrac{ee'}{r^2}$; and it was supposed in the old electrostatics that this formula is equally true whether the system is placed in a gravitational field or not. According to the theory of relativity, however, when the system is in a gravitational field, there is a distortion of space, the effect of which is that the ponderomotive force exerted electrically by P' on P is *not* equal and opposite to the ponderomotive force exerted by P on P'. The two forces therefore do not balance, but have a resultant which we may call F. Now let g be (to use for a moment the language of the older theory) the acceleration which the gravitational field produces on ordinary masses, and let us write $(F/g)=m$. Then what we have shown is that when the electrostatic system is placed in a gravitational field of acceleration g, a ponderomotive force mg acts on it, and therefore we may call m the "mass" of the electrostatic system. This elementary example shows very clearly how the idea of electric energy being subject to gravitation depends essentially on the idea of the distortion of space.

An interesting question now suggests itself. In this example we have shown that a system, which has no mass of the ordinary material kind, nevertheless behaves as if it had material mass, being subject to the attraction of a gravitational field. Is it possible that *all* mass is really of this non-material kind? In other words, is ordinary material mass not an independent physical entity, but merely an effect produced by a great concentration of electromagnetic energy? This is an attractive suggestion, but there is a difficulty which seems to bar the way; if I may use for a moment the language of the tensor-calculus,

the invariant $\sum_{p,q} g_{pq} E^{pq}$ of the electromagnetic energy-tensor is always zero, whereas the corresponding invariant $\sum_{p,q} g_{pq} R^{pq}$ of the material energy-tensor is always different from zero in the presence of matter; it seems therefore as if there is an essential difference between electromagnetic phenomena on the one hand and material phenomena on the other. But I do not think that this argument is conclusive; if ordinary "matter" represents only certain macroscopic aspects of particular kinds of electromagnetic systems, in which there are "kinematical constraints" (*e.g.* like that which prevents the charge of an electron from dispersing under the mutual repulsion of its parts), then it may be possible to construct electromagnetically an R^{pq} such that $\sum_{p,q} g_{pq} R^{pq}$ is different from zero in spite of its connexion with $\sum_{p,q} g_{pq} E^{pq}$ which vanishes.

I use the tensor-calculus reluctantly in this lecture, since a knowledge of it is by no means universal as yet, but I cannot quite avoid it in explaining the most profound of all the changes that have taken place of late years in the conception of energy. To introduce this, let us recall that energy is commonly regarded as a "scalar," *i.e.* an entity whose amount can be represented by a single number, which is the same number for all observers and for all axes of reference. Thus energy in its electrical form is sold to us by the electric-light companies at so much a unit, just as milk is sold at so much a quart; and this unit is a definite amount standardised by the Board of Trade. When we talk about "the whole amount of energy in a certain region" or "the whole amount of energy in the universe," we are tacitly presupposing that energy is a scalar, so that the amounts of it in different sub-regions may be added together to constitute the amount in the whole region. And yet a moment's reflection will show that this is not the full truth of the matter. Suppose, for instance, we enquire what is the kinetic energy of the sun's motion in space. It all depends on what we take the sun's velocity in space to be. We know that the sun has a certain velocity with respect to the aggregate of neighbouring stars, and that he has a different velocity with respect to the general body of the remoter stars, and yet a different velocity with respect to the star-clusters or the spiral nebulae. We cannot say that any of these velocities is his actual velocity in space, and we could not determine what that actual velocity is unless we had some absolutely fixed axes to refer it to. But, as the principle of relativity teaches us, no such set of axes exists; an infinite number of sets of axes can be found, each of which is as good as any other; and our estimate of the sun's velocity, and therefore of his kinetic energy, depends on the choice we make.

Let us, then, compare the measures of the kinetic energy of a moving body relative to two sets of axes $Oxyz$ and $O'x'y'z'$, when the axes $O'x'y'z'$ are moving relative to the axes $Oxyz$ in a direction parallel to the axes of x and x', with a velocity which we shall denote by $c \tanh a$, where c denotes the velocity of light and a is the constant which effectively specifies the velocity. For simplicity, suppose the body to be moving parallel to the axes of x and x'; then using the above expressions (2) and (3) for the momentum and kinetic energy, and remembering the formula for the composition of velocities in relativity, we have

$$\left.\begin{array}{l}(\text{kinetic energy of the particle in the system } O'x'y'z') \\ = \cosh a \times (\text{kinetic energy of the particle in the system } Oxyz) \\ - c \sinh a \times (\text{momentum of the particle in the system } Oxyz)\end{array}\right\} \quad \ldots\ldots(4)$$

Now let us find an equation with which this may be compared. Let (X, Y) be the components of an ordinary vector in a plane, referred to axes Oxy in the plane, and let (X', Y') be the components of the same vector referred to

another set of axes $Ox'y'$ in the plane, which make an angle a with the axes Oxy; then we have the elementary formula

$$X' = X \cos a - Y \sin a. \qquad \text{.................................}(5)$$

Now the equation (4) is really of the same nature as this equation (5), and what it shows is that kinetic energy is not a scalar, but is one component of a *tensor* (which is a kind of glorified vector), of which the components of momentum are also components. This tensor is called the *energy-tensor*, and it plays a great part in the new physics; quite as great a part as the scalar energy did in the old physics. Let us see to what extent the part is a similar one.

First of all we notice one great difference. Since energy is no longer a scalar, we cannot any longer divide a region into small sub-regions, calculate the amount of energy in each sub-region, and add these amounts together, in order to obtain the amount of energy in the whole region; for the energies in the sub-regions are different in kind, like components of momentum in different directions, and so cannot be added together. In fact, "the total amount of energy in a finite region" is a phrase which no longer has a meaning in the space of general relativity.

What, then, happens to the great doctrine of the Conservation of Energy? The old principle of conservation of scalar energy is no longer true, but an interesting and beautiful theorem (I speak as a mathematician) takes its place. As I explained earlier, a theorem of conservation is always represented mathematically by the vanishing of a divergence; and the new theorem is that *the divergence of the energy-tensor is null.* This assertion indeed is equivalent to four equations, of which one corresponds to the old equation of conservation of energy, and the other three to the old equations of conservation of momentum.

The new physics is more powerful and more comprehensive than the old, and above all it is more true, so let us study and enjoy it; but we may perhaps be pardoned for some sentimental regret that energy has lost the status of thinghood, and now counts as only one of the ten components of a symmetrical tensor of the second rank in the distorted manifold of space and time.

E. T. W.

GLEANINGS FAR AND NEAR.

653. A little instruction and guidance in science is sufficient for the intelligent student, for this alone will help him to develop his knowledge of his own accord. Science instilled into the intelligent mind has sufficient vitality in it to grow and expand by its own force even as a drop of oil on a sheet of water, a piece of secret confided to a villain, or a little act of charity to the deserving person.—Quoted from *Bhaskara* by Mr. A. A. K. Ayyangar. *Notes and Questions*, p. 2. *Journal of the Indian Mathematical Society*, Feb. 1929, vol. xviii. No. 1.

654. As the prerogative of Natural Science is to cultivate a taste for observation, so that of Mathematics is, almost from the starting point, to stimulate the faculty of invention.—J. J. Sylvester, *Nature*, i. p. 261.

655. "This Prince (Abdulla Qutb Shah, King of Golkonda, 1611-72) passionately loves all those who are proficient in mathematics, which he understands fairly well; hence, although he is a Musulman, he favours all Christians who are learned in this science, as he particularly showed in the case of the Rev. Father Ephraim, a Capuchin. . . . He did all he could to induce him to remain in the country, and offered to build for him, at his own cost, a house and a church. . . ."—Tavernier, *Travels in India*. Translated by V. Ball. Second edition, Oxford, 1925, vol. i. p. 132.

For the further history of Father Ephraim, cf. *ibid.* pp. 176-86. [Per Mr. Puryer White.]

DIMENSIONS AND IDENTITY OF VECTOR DIRECTION.*

BY PROF. A. LODGE, M.A.

IN an equation with real coefficients, if the terms are vectors, they must be codirectional if the equation is possible with all the quantities real. Thus to obtain a geometrical solution of the equation $a\cos\theta + b\sin\theta = c$, a and b must be drawn at right angles to each other since $a\cos\theta$ makes an angle θ with a, and $b\sin\theta$ makes an angle $90° - \theta$ with b. The solution is obtained by the intersections of the circles $r = a\cos\theta + b\sin\theta$, $r = c$.

Consider the expression $ax+by+c$ which is proportional to the distance of P, (x, y), from the line $ax+by+c=0$. As it stands its exact meaning is difficult to arrive at, but if we divide by a we obtain $x+\frac{by}{a}+\frac{c}{a}$, in which every term must be in the x direction: it therefore is the distance of P from the line, measured in that direction. It is easy to specify the geometrical meaning of each term: consider each term to be positive.

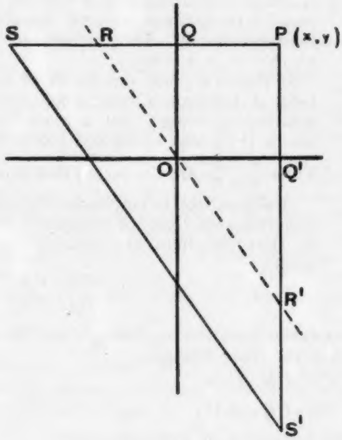


FIG. 1.

Going from P, from right to left, $PQ = x$, QR carries us the distance $\frac{by}{a}$ to a point Q on the line OQ parallel to the required line, as is seen by imagining $c=0$.

The final lap $\frac{c}{a}$ carries on to the line itself, this length being equal to the parallel distance of the line behind the origin.

Similarly, $PQ'R'S'$, viz. $\frac{ax+by+c}{b}$, denotes the distance of P from the line, measured in the y direction, from above downwards. [If any of the coefficients are negative, the corresponding lengths must be measured backwards.]

* Extracts from, and additions to, an address given to the London Branch on Feb. 2, 1920.

Calling these distances X, Y, then if the axes are rectangular and p is the perpendicular distance of p from the line, the equation $\frac{1}{p^2}=\frac{1}{X^2}+\frac{1}{Y^2}$ leads at once to

$$p=\frac{ax+by+c}{\sqrt{(a^2+b^2)}};$$

positive for points P to the right of the line if a is positive,
" " " above the line if b is positive.

The above analysis enables a line to be drawn without the trouble of finding points on it. For example take the line $2x+3y+4=0$.

It will be convenient to fix on a point P whose ordinate is 2, so that $\frac{by}{a}$, *i.e.* $\frac{3y}{2}$, is equal to 3.

Taking the horizontal distance $x+\frac{3y}{2}+2$: x takes us to Q, 3 more takes to R on the parallel line through O, so fixing the direction, and 2 more brings us to the line itself, which is to be drawn parallel to OR.

Examples of scalar equations.—The squares of vectors, and the product of parallel or collinear vectors, are scalars, and need not be codirectional. The simplest case is "Pythagoras'": $a^2=b^2+c^2$ if A is 90°.

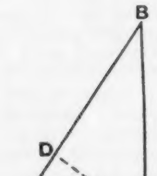


FIG. 2.

In Euclid's proof, one factor of each term is forcibly twisted through a right angle, and the proof is by equality of areas; but a more fundamental method leaves the terms alone and solves by similar triangles. Thus $\begin{matrix}CDA\\CAB\end{matrix}$ are similar, *in that order of angular points.*

(I should like to emphasise the importance of always specifying similarity of triangles in this orderly manner, as that simplifies the picking out of corresponding sides.)

$$\therefore \frac{CD}{CA}=\frac{CA}{CB},$$

the numerators being taken from the top triangle and the denominators from corresponding places in the lower triangle;

$$\therefore CD \,.\, CB=CA^2.$$

Similarly $\quad DB \,.\, CB=BA^2; \quad \therefore$ etc.

In like manner, the extension of Pythagoras can be obtained by similar triangles.

$\begin{matrix}CDA\\CEB\end{matrix}$ are similar;

$$\therefore \frac{CD}{CE}=\frac{CA}{CB};$$

$$\therefore CD \,.\, CB=CA \,.\, CE$$
$$=b\,(b-c \,.\, \cos A).$$

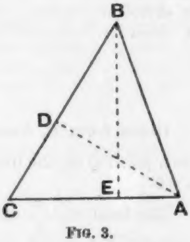


FIG. 3.

Similarly, by drawing the third perpendicular,

$$DB \,.\, CB=c\,(c-b \,.\, \cos A);$$

$$\therefore a^2=b^2+c^2-2bc\cos A,$$

a scalar equation, always used for numerical purposes, with no desire to think of areas.

Mechanical formulae.—These have been dwelt on at various times, so they may be dismissed briefly.

Such a formula as $v^2 \div 2g$, when v is a vertical velocity and g the vertical acceleration due to gravity, is interesting. Its directional dimensions are $\frac{Y^2}{T^2} \div \frac{Y}{T^2} = Y$. Similarly $\frac{v}{g} = \frac{Y}{T} \div \frac{Y}{T^2} = T$.

The formulae for the horizontal range of a projectile, or for its range on an inclined plane, are full of interest: they were discussed at some length in the *Gazette* for December 1925, pp. 503-505.

The expression indicating hardness of material given in some books is $\frac{vpat}{m}$, where two surfaces of area a are rubbed together with velocity v for a time t under an intensity of pressure p, during which process a mass m is rubbed off. Here pa is a force, vt a distance, and at first sight the formula seems to give the work required to rub off a mass m. But surely if p is the normal pressure, μpa is the force in the direction of vt, and if p is the tangential force it would seem more natural to replace pa by a single letter F. At any rate the formula lends itself to criticism on analysis.

Most mechanical and physical formulae are worth having their structure analysed dimensionally.

The same is true of formulae in the differential calculus.

Thus if x, y are rectangular coordinates of a moving point in a plane, $\frac{dy}{dx}$ is of dimensions $Y \div X$, *i.e.* of the tangent of an angle; $\frac{d^2y}{dx^2}$ is of dimensions $Y \div X^2$, or, if we replace the scalar X^2 by Y^2, it is of dimensions Y^{-1}, and measures the curvature of the path at points when y is a maximum or minimum: though for other points it requires multiplication by $\cos^3 \phi$ (where $\tan \phi = dy/dx$), which turns it from a vertical vector into a vector along the normal to the path of the point, its reciprocal being the radius of curvature.

So also partial differentiation formulae repay careful analysis. *E.g.* if a function of x and y changes from z to $z + \delta z$ when x changes from x to $x + \delta x$, y unchanged, and then y changes from y to $y + \delta y$ on returning to the former value, z, of the function. The two changes of the function add up to zero:

i.e.
$$\frac{dz}{dx}\delta x + \frac{dz}{dy}\delta y = 0.$$

Also to find the steepest gradient of the function, which we may think of as denoting altitudes at points of a map: let δn be the normal distance between two contour lines z, $z + \delta z$.

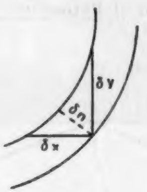


FIG. 4.

Then $\frac{1}{\delta n^2} = \frac{1}{\delta x^2} + \frac{1}{\delta y^2}$ in the limit when PR may be considered straight.

Multiplying through by $(\delta z)^2$, and proceeding to the limit

$$\left(\frac{dz}{dn}\right)^2 = \left(\frac{dz}{dx}\right)^2 + \left(\frac{dz}{dy}\right)^2,$$

the square root of this quantity being the required gradient, the geometry being quite elementary.

Geometrical solution of the quadratic equation $x^2-bx+ac=0$. The roots of this equation are the abscissae of the points where the system of coaxial circles $x^2+y^2-bx-2ky+ac=0$ cut their common radical axis $y=0$. There is no difficulty in drawing any one of these circles: thus, let OB on the x-axis $=b$, and draw a line through O, marking on it the distances OA, $OC=a$, c respectively. Then the perpendicular bisectors of OB, AC meet at the centre, and AC is a chord, of one of these circles.

But suppose these circles refuse to reach the x-axis, what becomes of the construction now? Does it fail? It does if we refuse to leave the axis, but that is our own doing: the solution is there just the same if you allow x to leave its own axis and trench on the domain of y. In fact the coaxial system $x^2+y^2-bx-2ky+ac=0$ now has in itself two point circles obtained by writing the equation in the form

$$\left(x-\frac{b}{2}\right)^2+(y-k)^2=k^2+\frac{b^2}{4}-ac.$$

and making
$$k^2+\frac{b^2}{4}=ac.$$

Call these points P, Q; their coordinates are $\left(\frac{b}{2}, k\right)$ and the roots are

$$\overline{OP}=\tfrac{1}{2}b+ik,$$
$$\overline{OQ}=\tfrac{1}{2}b-ik,$$

where
$$k^2+\tfrac{1}{4}b^2=ac;$$

so that the actual lengths of these vectors are each $\sqrt{(ac)}$, which is the tangential distance of O from any circle of the system.

The product $\overline{OP}\,.\,\overline{OQ}$ is equal to

$$(\tfrac{1}{2}b+ik)(\tfrac{1}{2}b-ik)=\tfrac{1}{4}b^2+k^2=ac;$$

$$\therefore \text{ if } \overline{OP} \text{ is } x,\ \overline{OQ} \text{ is } \frac{ac}{x},$$

and it is evident that

$$\overline{OP}+\overline{OQ}=\overline{OP}+\overline{PB}=b,$$

i.e.
$$x+\frac{ac}{x}=b;$$

$$\therefore x^2-bx+ac=0,$$

all very simple and satisfactory.

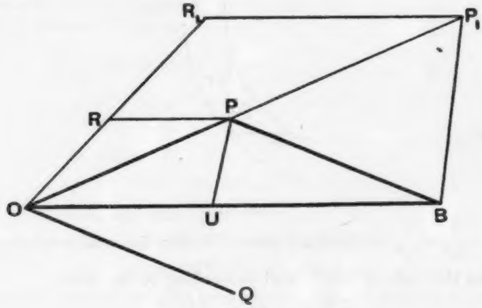
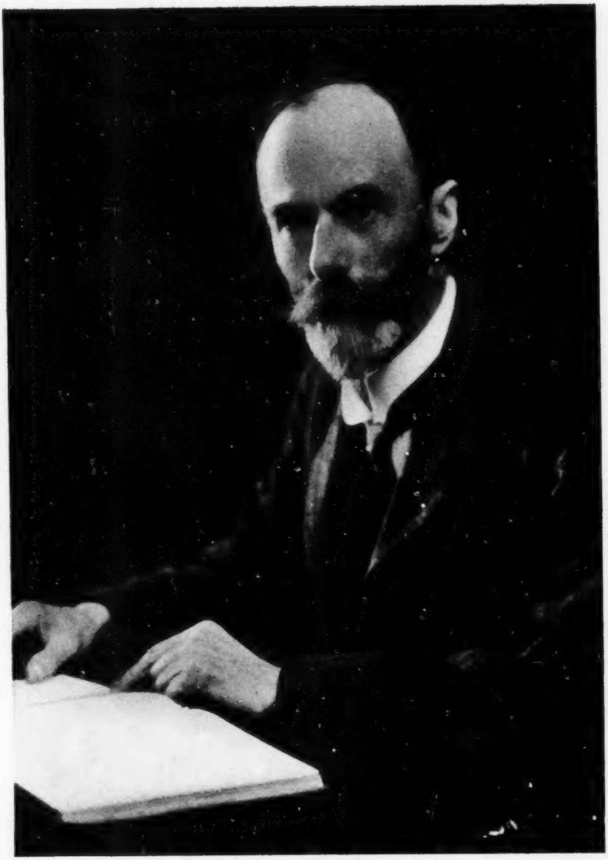


FIG. 5.



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F. S. MACAULAY.

Now comes the question, what is x^2? We shall have to traverse our previous careless statement that x being a vector, x^2 is scalar. That was true when x was a mere multiple of some unit along its own line, but is no longer true when x has to rotate out of its own line. First write the equation in the form $\frac{x^2}{b}+\frac{ac}{b}=x$.

Now the ratio of $\frac{x^2}{b}$ to x is equal to the ratio of x to b. This involves similar triangles as well as proportional lengths. Hence, on OP construct an isosceles triangle ORP similar to OPB. Now $\overline{OR}=x^2/b$, $RP=\frac{ac}{b}$ being parallel to OB, and $\overline{OR}+RP=\overline{OP}$, *i.e.* $\frac{x^2}{b}+\frac{ac}{b}=x$.

Finally, let OU denote unit length along OB, and magnify the triangle OPR till the sides are increased in the ratio $OB:OU$, *i.e.* each side is b times its previous length. Let OP_1R_1 be this new triangle.

Then $\qquad \overline{OR_1}=x^2,\ R_1P_1=ac,\ \overline{OP_1}=bx,$

and therefore, evidently, $\qquad x^2+ac=bx.$

N.B.— $\qquad \overline{OR_1}$ is really $\frac{x^2}{\overline{OU}}$, $R_1P_1=\frac{ac}{\overline{OU}}$, and $\overline{OP_1}=\frac{bx}{\overline{OU}}$,

i.e. they are of the dimensions of length really. It is only by the artificial idea of calling OU unity that we spoil *dimensional* tests. But even this does not spoil *directional* necessities so long as we measure OU along the simple x-axis itself.

Images in a line.—The fact that the points P, Q are images of each other in the radical axis of the circles of which P, Q are the limiting points suggests a quick method of finding the image of any point (x', y') with regard to a line $ax+by+c=0$.

The system of coaxial circles is

$$(x-x')^2+(y-y')^2=2k\,(ax+by+c),$$

$$i.e.\ (x-x'-ka)^2+(y-y'-kb)^2=k^2(a^2+b^2)+2k\,(ax'+by'+c);$$

$\therefore$ putting the right-hand side = zero, the image is at

$$(x'+ka,\ y'+kb)\quad \text{where}\quad k=-\frac{2(ax'+by'+c)}{a^2+b^2}.$$

This can be verified vectorially.

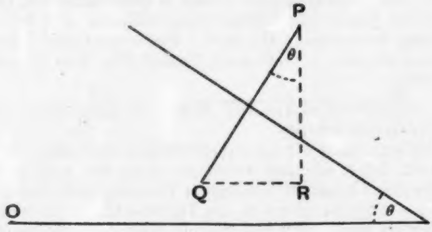

FIG. 6.

In the diagram $\qquad \overline{OQ}=\overline{OP}-\overline{RP}-\overline{QR}$

$$=(x'-2p\sin\theta)+i\,(y'-2p\cos\theta)$$

where $\qquad p=\frac{ax'+by'+c}{\sqrt{a^2+b^2}};\ \tan\theta=\frac{a}{b};$

$$\therefore\ \sin\theta=\frac{a}{\sqrt{a^2+b^2}}\,;\quad \cos\theta=\frac{b}{\sqrt{a^2+b^2}}\,;$$

$$\therefore\ 2p\sin\theta=-ka\,;\quad 2p\cos\theta=-kb.$$

Signs are a bit awkward by this method, but there is no difficulty in the coaxial circle method. An example may make this clear.

Find the image of $(3, 4)$ in the line $4x-6y-1=0$.

The coaxial system is

$$(x-3)^2+(y-4)^2=2k\,(4x-6y-1),$$

i.e.
$$(x-3-4k)^2+(y-4+6k)^2=52k^2-26k$$
$$=0 \text{ if } k=0 \text{ or } \tfrac{1}{2}\,;$$

$\therefore$ the image is at $(3+4k, 4-6k)$, where $k=\frac{1}{2}$,

i.e. at $(5, 1)$.

This example was chosen to give a neat value to k, but the method is equally simple whatever k may be, as we merely have to substitute its value at the end.

ALFRED LODGE.

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656. J. M. Barrie, in his brightly written *An Edinburgh Eleven*, calls [Thomson and Tait's *Elements of Natural Philosophy*] " the Student's first glimpse of Hades."—Knott's *Life and Scientific Work of P. G. Tait*, 1911, p. 201.

657. Sur tout ne vous laissez point ensorceller par les atraits diaboliques de la Géométrie : rien n'éteindrait tant en vous l'esprit intérieur de grace. . . .—Fénélon, *Lettres Spirituelles*, Amsterdam, 1723 (Letter 148).

658. Les quantités irrationelles ne sont pas grandeurs inexpliables. (After applying Pythagoras' Theorem to find the hypotenuse $AC=\sqrt{41}$, where $AB=4$ and $BC=5$, he proceeds :) et ceste operation est plus commode que par Algebre, en laquelle l'un singe et faict la mine que le notoire soit incogneu, ce qui est inutile.—*Les Œuvres Mathematiques de Simon Stevin.* Girard's Edition. Leyden 1684.

659. There can be no doubt about faith and not reason being the *ultima ratio.*

Even Euclid, who has laid himself as little open to the charge of credulity as any writer who ever lived, cannot get beyond this. He has no demonstrable first premise. . . . His superstructure indeed is demonstration, but his ground is faith. Nor again can he get further than telling a man he is a fool if he persists in differing from him. He says " which is absurd " and declines to discuss the matter further. . . .—Samuel Butler, *The Way of All Flesh*, c. 65, p. 312 (Nash, 1928).

660. From a description of the *Tāj Mahal* by Manrique, a missionary in India in the seventeenth century.

This great wall embraced a huge square-shaped enclosure, in the centre of which rose a vast, lofty, circular structure, from the middle of which this famous Geometer (*i.e.* Geronimo Veroneo, a Venetian goldsmith, who, according to Manrique, was the architect of the Tāj Mahal), by drawing equal lines, constructed a perfect circle with less trouble than Archimedes of Syracuse.—*Travels of Manrique,* edited by Luard (Hakluyt Society, 1927), ii. 172. [Per Mr. Puryer White.]

661. Numero deus impare gaudet.—Verg. *Ecl.* 8, 75.

662. This is the third time ; I hope good luck lies in odd numbers. . . . They say, there is divinity in odd numbers, either in nativity, chance, or death.—*The Merry Wives of Windsor,* v. 1.

NOTE ON CARTESIAN GEOMETRY.

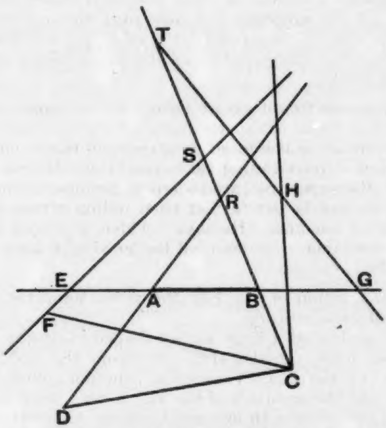
By The Rev. J. J. Milne, M.A.

When it is said that Descartes was the inventor of analytical geometry, the impression is often given that by Cartesian Geometry is simply meant a system in which the position of a point is determined by means of its coordinates referred to two fixed straight lines, either at right angles or oblique. A little consideration shows us that it is not correct to say that this system was invented by Descartes, as we find that Apollonius, Bk. V, Prop. 52, gives the values of the coordinates of the centre of curvature of a point on a conic referred to two rectangular axes. It will no doubt interest the readers of the *Mathematical Gazette* to know what is was that Descartes invented, and what led him to it.

In Cap. 2 of his *Geometry* Descartes quotes a passage from Pappus respecting the problem generally referred to as the *locus ad tres et quatuor lineas*, which Pappus said that neither Euclid nor Apollonius nor anyone else had been able to completely solve. Taking the case in which four straight lines are given, the problem may be stated as follows:

If straight lines are drawn from a moving point making given angles with four given straight lines, and if the rectangle contained by two of the lines so drawn bears a given ratio to the rectangle contained by the other two lines, it is required to find the locus of the moving point. Descartes in his letters, *Cousin*, vol. vi. pp. 224 and 294, tells us that he arrived at the solution of this problem four years before the publication of his *Geometry* in 1637, after spending five or six weeks on it, and in the first two chapters he employs this problem to develop his method of analytical geometry as follows:

Let AB, AD, EF, GH be four straight lines given in position, and let it be required to find the locus of a point C from which straight lines CB, CD,



CF, CH are drawn making given angles CBA, CDA, CFE, CHG respectively with the given lines, and such that the product of two of them has a given ratio to the product of the other two.

Descartes considers one of the given lines AB, and CB one of the lines through C as principal lines to which he refers all the others.

Let $AB=x$, and $BC=y$, and produce the other given lines AD, EF, GH

to meet the principal lines in the points A, R; E, S; and G, T. Let $AE=k$, $AG=l$.

Then since all the angles of the triangle ARB are known, the ratio AB to BR is known.

Let $AB : BR = z : b$, where z and b are known constants;

$$\therefore\ RB=\frac{bx}{z};\quad \therefore\ CR=y+\frac{bx}{z}.$$

Similarly in the triangle CDR, let $CR : CD = z : c$;

$$\therefore\ CD=\frac{c}{z}\,.\,CR=\frac{cy}{z}+\frac{bcx}{z^2}.$$

In the triangle ESB let $BE : BS = z : d$;

$$\therefore\ BS=\frac{d}{z}\,.\,BE=\frac{d(k+x)}{z};$$

$$\therefore\ CS=y+BS=\frac{yz+d\,(k+x)}{z}.$$

In the triangle FSC let $CS : CF = z : e$;

$$\therefore\ CF=\frac{e}{z}\,.\,CS=\frac{ezy+ed\,(k+x)}{z^2}.$$

In the triangle BGT, let $BG : BT = z : f$;

$$\therefore\ BT=\frac{f}{z}\,.\,BG=\frac{f}{z}(l-x);$$

$$\therefore\ CT=y+BT=\frac{zy+f(l-x)}{z}.$$

In the triangle TCH, let $CT : CH = z : g$;

$$CH=\frac{g}{z}\,CT=\frac{gyz+fg(l-x)}{z^2}.$$

Thus the length of each of the lines CB, CD, CF, CH can be expressed in the form $ax+by+c$, where a, b, c are known constants. To determine the point C we have the condition $CB\,.\,CF=\lambda\ CD\,.\,CH$ where λ is the given ratio Giving to CB, etc., their values in the form $a_1x+b_1y+c_1$ the condition can be expressed as an indeterminate equation of the second degree in the two unknown quantities, x and y, and giving any value we please to either x or y we can find the value of the other from the resulting equation. He then proceeds to say that all points of a geometric curve must bear a definite relation to the points on a straight line, which relation can be expressed by a single equation. Apollonius had shown that an equation of the first degree represents a straight line, and Descartes showed that the equation of the second degree includes the circle, parabola, ellipse and hyperbola according to the relative values of the constants.

In the particular instance given in the figure by assigning to the given quantities the following numerical values, $EA=3$, $AG=5$, $AB=BR$, $BS=\frac{1}{2}BE$, $GB=BT$, $CD=\frac{3}{2}CR$, $CF=2CS$, $CH=\frac{2}{3}CT$, the angle $ABR=60°$, $\lambda=1$, he obtains the equation $y^2=2y-xy+5x-x^2$, and after further examination into which we need not follow him he shows that the locus of C is a circle.

JOHN J. MILNE.

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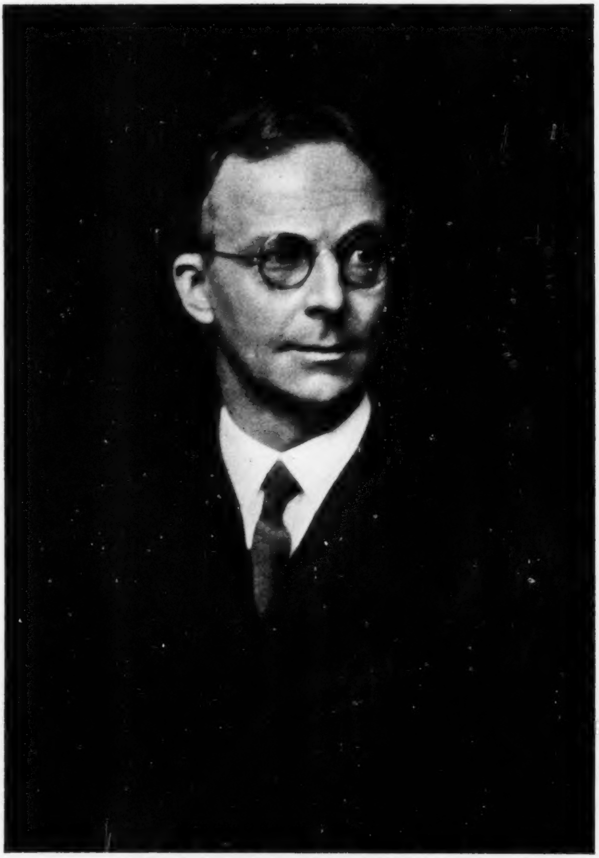
663. "Some Performances of this Kind have already appeared, one of which was stifled in its Infancy, by having been nursed in the Country, where proper Support and Encouragement were not to be had; others yet exist in the Land of Literature, but so deformed, mutilated, and monstrous, through the Want of Ability in their Parents and Guardians, that their End is defeated, the Publick receiving no Improvement or Emolument thereby."—Preface to *The Mathematician*, 1745.



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PROF. E. T. WHITTAKER.

SIR ISAAC NEWTON'S EDITION OF VAREN'S GEOGRAPHY.

By Prof. Florian Cajori, Ph.D., University of California.

The belief entertained by some writers that Newton's edition of Bernhard Varen's *Geographia generalis* was simply a verbatim reprint of the original Amsterdam edition of 1650, or of 1664 or 1671, which did not demand Newton's thoughtful attention, is erroneous. When the first edition went through the press, Varen is said to have been in a bad state of health,* which would account for certain gross errors in the text which needed correction, and for the absence of geometric figures which the reader finds it irksome to supply from the statements in the text. The editions of 1650 and 1671 contain only three figures. In Newton's edition (1672) these are redrawn in improved form and thirty other geometric figures are added. This fact alone indicates that Newton read the text critically. That these drawings are really Newton's own work is evident from his remark in a letter of May 25, 1672, to Collins: † "The book here in the press is Varenius his Geography, for which I have prepared schemes." We have compared Newton's edition with the original, in Chapters IV. and IX., and have found minute alterations on nearly every page. Thus Newton changes "Itaque ut 7 gr. 30 min. ad 1 gr. sive ut $\frac{1}{48}$ ad $\frac{1}{360}$, hoc est, ut 48 ad 360 ita 5000 ad $666\frac{2}{3}$ stadia" into "Itaque ut 7 gr. 30 min. ad 1 gr. sive ut $\frac{1}{48}$ ad $\frac{1}{360}$ hoc est, ut 360 ad 48 ita 5000 ad $666\frac{2}{3}$ stadia." ‡ When Varen says that the moon is distant 40 radii of the earth, Newton changes 40 to 59.§

Varen gives Snell's measurement of a degree of latitude to be "28500 perticarum ... hae faciunt $19\frac{147}{180}$ mill." Newton observed the error in expressing perches in Dutch miles, and made a correction so as to read, "28500 perticarum ... hae faciunt $18\frac{147}{180}$ mill." ‖ The table of distances at which a mountain peak can be seen at sea (disregarding refraction), given by Varen is erroneous, and is corrected by Newton. Thus a mountain height of one German mile could be seen at a distance of $29\frac{1}{4}$ German miles according to Varen, and $41\frac{1}{2}$ German miles according to Newton.¶

Chapter IV., in Varen's book, on the size of the earth, is of importance to readers interested in the cause of Newton's twenty years' delay in announcing the law of gravitation. From descriptions in the text, Newton drew three geometric figures which are used in the explanation of each of the seven methods of determining the size of the earth, described by Varen.** The measurement due to Snell receives Varen's special attention and is critically discussed. Snell obtains per degree of latitude $18\frac{147}{180}$ (nearly 19) Hollandish miles. Newton's correction of the error "$19\frac{147}{180}$," noted above, is conclusive proof that he had familiarized himself with the results of Snell's meridian measurement. If further evidence on this point were necessary, we could cite Varen's statement regarding German miles, "quorum XV faciant XIX milliaria Hollandica" †† and Newton's use of Snell's value for the degree in computing (as stated above) the distance of visibility at sea from an elevation of one German mile, and also of certain fractions thereof. There is evidence ‡‡

* S. Günther, *Varenius*, Leipzig, 1905, pp. 27, 160.

† S. P. Rigaud, *Correspondence of Scientific Men of the Seventeenth Century*, Oxford, vol. 2, 1841, pp. 322, 335.

‡ Varenius, *Geographia generalis*, ed. of 1650, p. 38; ed. 1671, p. 36; ed. 1672, p. 26. The edition of 1664 we have not seen. Later editions of Newton's revision appeared in 1681, and, in English translation, in 1733, 1734, etc.

§ Edition 1650, p. 44; ed. 1671, p. 42; ed. 1672, p. 30.

‖ Edition 1650, p. 39; ed. 1671, p. 37; ed. 1672, p. 27.

¶ Edition 1650, p. 89; ed. 1671, p. 85; ed. 1672, p. 61.

** Edition 1650, pp. 30-47; ed. 1671, pp. 29-45; ed. 1672, pp. 21-32.

†† Edition 1650, p. 42; ed. 1671, p. 40; ed. 1672, p. 29.

‡‡ See *Sir Isaac Newton* (1727-1927), Baltimore, 1928, pp. 162, 164.

that Newton probably knew Snell's value in 1666, when he made his first earth-moon test of the law of gravitation. The evidence in Varen's book makes it certain that Newton knew that value in 1672. On no other occasion, previous to his writing the *Principia*, is it known definitely what meridian value Newton actually used. This value would yield a fairly satisfactory agreement between computed and observed values in the earth-moon test. The data here presented constitute a strong and additional argument against the old theory that Newton's delay of twenty years, in announcing the law of gravitation, was due to his use of too small a value for the radius of the earth.

University of California. FLORIAN CAJORI.

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664. "Let us agree to call the products

$$(1-2a\cos\theta+a^2)(1-2a\cos 3\theta+a^2)\ldots(1-2a\cos(i+1)\theta+a^2),$$

$$(1-a)(1-2a\cos 2\theta+a^2)(1-2a\cos 4\theta+a^2)\ldots(1-2a\cos 2a\theta+a^2),$$

cascades of degrees $2i+1$, $2i$ respectively, a being called the parameter and θ the argument.—*Sylvester*, in letter to W. J. C. Miller, Oct. 11, 1877.

Rolle's cascades were formed as follows (v. *Traité d'Algèbre*, 1690, p. 125):

Taking an equation such as

$$4x^4-17x^3+63x^2-37x+194=0, \qquad (1)$$

he forms another by multiplying each term by the power of its x, and rejecting x he gets

$$16x^3-51x^2+126x-37=0 \qquad (2)$$

Similarly he finds, forming as before and dividing by $2x$,

$$24x^2-51x+63, \quad 48x^2-102x+126=0, \qquad (3)$$

and again, but dividing by $3x$,

$$32x-34=0. \qquad (4)$$

Here (4), (3), (2), (1) are the 1st, 2nd, 3rd, 4th cascades respectively.

Cf. *Gleanings*, No. 9.

665. *An Astronomical Catechism*, 1818, by Catherine Vale Whitwell.

There was . . . one who paid great attention to those nightly changes in the moon . . . and from this simple fact arose the fiction of the planet being quite in love with the man.

He must, I think, have been pleased with this: p. 9.

Will it not be interesting . . . to learn what could have occasioned the neglect of such a delightful subject?

Yes, that would be very agreeable. p. 22.

. . . is there really any thing in the Bible which ought to have puzzled him? p. 27.

Who are the persons who have distinguished themselves by their careful construction of telescopes, and by their exposure to the cold midnight air. p. 33.

666. Samuel Juxon to Sir Thomas Smythe, Governor of the East India Company, March 2, 1614-5. From Swally Road (India).

. . . First, for the mathematical science, I acknowledge my imbecility such as in the best sufficiency that I am able therein I can give the Right Honourable and Right Worshipful Company small or no content. . . .—*Letters received by The East India Company*, ed. Foster, vol. iii. (1899), p. 35.

The Editor comments on this letter: "Samuel Juxon was apparently quite a young merchant. His letter is written in a beautiful hand, but its diction is scarcely to be commended and its matter is very justly summed up in the endorsement 'Of no moment at all.'"

SIR GEORGE GREENHILL, F.R.S.

BORN NOV. 29, 1847—DIED FEB. 10, 1927.

IN an attempt to give a permanent impression of a unique personality in the ranks of mathematics,—a personality whose like will certainly not be seen again,—for he lived, essentially, in the days which produced some of our most revered pioneers born outside their own appropriate age,—Augustus de Morgan was born too early, perhaps, and Greenhill too late,—we may give, in his own words, a succinct summary of his own impression of his career. It is modest to a degree.

George Alfred Greenhill was educated at Christ's Hospital and St. John's College, Cambridge. He became a Fellow of his College, and spent the greater part of his life, before retirement, as Professor of Mathematics in the Artillery College at Woolwich. He was knighted on retirement in 1908, and subsequently lived in famous rooms at 1 Staple Inn, W.C., pursuing mathematical researches. His publications include the following:

1885. Differential and Integral Calculus, with Applications.
1892. Applications of the Elliptic Function.
1894. Hydrostatics.
1908. Notes on Dynamics.
1910 (and again 1916). Report 19, on Theory of a Streamline, with Applications to an Aeroplane.
1912. The Dynamics of Mechanical Flight.
1914. (Report 146.) Gyroscopic Theory.

The above account is not exactly his statement as made in books of reference which besieged him, but it is nearly so. When he had given this, he was tired, for publicity of any kind was obnoxious to him. This fact, however, renders it difficult for anyone in the present generation to obtain more precise information. As it would be absurd to make any attempt at concealment of the identity of the writer of this notice, he will at once say that he was Greenhill's neighbour for many years at 1 Staple Inn, and a lifelong friend after he took his own degree. Any account of Greenhill must necessarily be of an anecdotal form, and from multitudinous letters which the writer has seen, at one time or another, but will not quote with names and context, he feels that he has almost the whole Royal Artillery, of a certain generation, behind all he says. For Greenhill was loved by his old pupils to a degree which few Professors can have enjoyed. We shall, however, now refer back to his account of his career, for the omissions are serious, if characteristic. The value of his work, in a mathematical sense, is not at the moment in question.

His baptismal name was George Alfred Greenhill, but he subsequently reversed the order of his Christian names. Well-authenticated rumour has it that the reason was his dislike of a nickname the others led to in his youth. The best authentication is perhaps provided by the fact,—practically unknown even to his closest friends, and for which I am indebted to our mutual friend, Professor Ernest Wilson,—that he was once a Whitworth Scholar. This must be a considerable surprise to most mathematicians, but the entry in the "Whitworth book" is as follows:

"Greenhill, Sir George, Kt., M.A.; Wh. Sc. 1869; b. 1847. Formerly Professor of Mathematics in the Artillery College, Woolwich. Author of the following works:" (These need no repetition.)

But there is a further entry, and under the heading "Scholars appointed in 1869", we read:

"Greenhill, George A., (21) Student, Cambridge University.
"Hopkinson, John, B.Sc., (19) Student, Cambridge University."

It is of interest that these two pioneers, one ultimately becoming a mathematician with a strong practical bent, and the other a great engineer with a

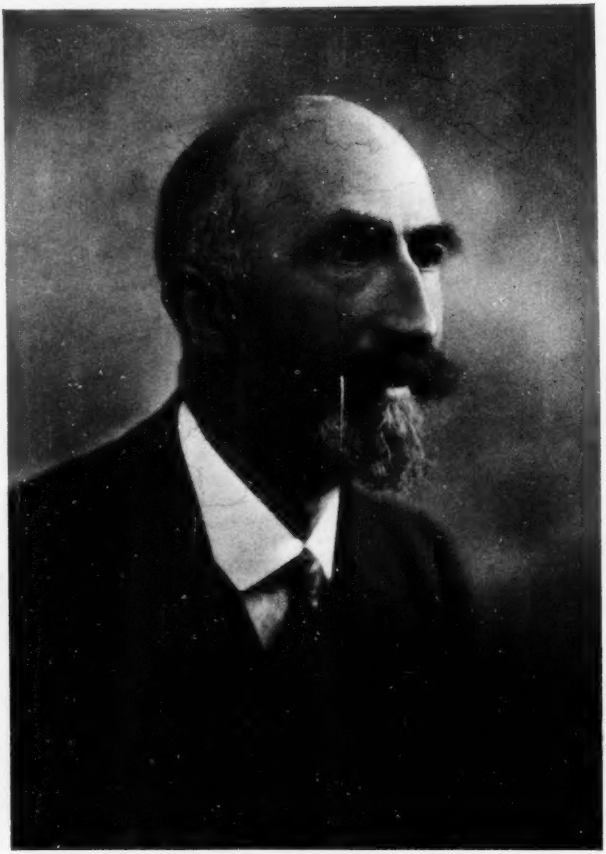
strong mathematical bent, should have met on this basis. May we conjecture, as all must have done who have studied such works of Greenhill as the Treatise on Elliptic Functions, that at this time he was at the cross-roads where the practical engineer and the mathematician take their leave of each other ?—but neither able to cast off the old love entirely, or to resist the colour it gave to all their subsequent work. In this, at least, I think we have the explanation of the very distinctive style which marks off all work done by Greenhill from work by others,—more professedly mathematicians,—in the same spheres.

After an intensely intricate and perhaps laborious calculation in regard to elliptic functions, no mathematician is unaware of the difficulty of some of these, and the uncanny facility which Greenhill had in their performance, he suddenly became equally intensely practical, and regarded his result as of no real value until its correspondence with phenomena shown by some inorganic "corpus vile" had been investigated. This inclination is perhaps specially pronounced as regards his work on elliptic functions, but it went through all he did, and, to him, "practical", or "applicable", or even "useful", was determined by some activity of man against the inorganic corpus. Perhaps two historical instances may suffice. Once the writer and others were candidates at an examination. He wished to set what was, in effect, a perfectly reasonable example on the catenary. He set it as a problem (two-dimensional) of "flexible inextensible sailcloth",—terrifying nearly all the candidates. Later, again in reference to the catenary, he gave an address, on a well-remembered evening, to the members of the Cambridge Mathematical and Physical Society on "Catenaries". It was probably as brilliant an address, from any mathematical standpoint, as they ever had, but he combined it with a kind of conjuring entertainment. At intervals, from all kinds of remote pockets, he produced objects to illustrate his remarks, especially in regard to the unstated assumptions in all current books of the time,—and, incidentally, of this time, perhaps,—and in regard also to his distrust of the "idealised" problems of applied mathematics forced upon the student. He produced, for example, from nowhere apparently, a long piece of thick and stiff rope, and held it up, with the supports at the same level. Its shape is left to the imagination, but he said, "This is the common catenary",—and proceeded to the next section of his address.

We now turn again to the omissions in his autobiography. A brief summary, again, will serve, and is not meant to be complete. He was elected to the Royal Society in 1888, served on its Council from 1896 to 1899, and was a Royal Medallist of the Society. Many foreign distinctions came to him. We mention, in particular, that he was Officier d'Académie de Paris, Corresponding Member of the Academy of Sciences of Paris, and Foreign Member of the Reale Accademia des Lincei. He was almost an active member of some, for they had frequent visits from him, and he had a multitude of close friends among Continental mathematicians. He had played his part, as a great figure, in other Societies. He had been President of the London Mathematical Society, and much of his most valuable work is to be found in the publications of the American Mathematical Society.

A picture of such a figure as Greenhill, however, would only be blurred by any catalogue of his distinctions, or any attempt to appraise precisely his mathematical work in relation to that of others of his time. It is sufficiently well known, and his influence has been so profound on all subsequent workers, that none interested in the subject, whether from the tabular standpoint or any other, can be unaware of it. We shall only recall the fundamental originality of his treatment of problems in his reports published by the Stationery Office in his later years, for they all immediately became classical, and its needs no old Army pupil of his to understand the value they held at a very critical time in our history.

No fellow mathematician, and nobody else who had come much into contact



F. W. Hill.

with him, whatever his own private views, could fail to admire the inflexible purpose which he pursued when his mind was made up on any subject. The view of him, drilling, at his age, under a junior officer, in the Temple during the war, was one pathetic to some observers, depressing to others, and very stimulating to yet more, but surely an object of admiration to all. As our leading expert on any topic dealing with projectiles,—well are remembered his calculations while Big Bertha shelled Paris,—equal admiration must go to his efforts at this time to become a practical expert in musketry. It is an admiration of character, with claims on those of all views.

To some extent, the practical interests which inspired the lines of his main work in pure mathematics can be grouped under a small number of headings, which, however, give a most inadequate view of his profound contributions, either in outlook, mode of treatment, or results achieved, to subjects such as the theory of Elliptic Functions. They relate to, roughly, gyrostatic problems, problems of stability of ships and aeroplanes, apart from his more professional work on projectiles. They inspired, but did not bound, his achievement. He was a fine classical scholar, capable of sustaining a historical point with any opponent, if any scientific machine or a classical reference to one was in question. At the same time he could hold his own on a philological point, for classics constituted perhaps his main hobby.

He showed his friends, with delight, photographs and drawings of all historical attempts to evolve flying machines, navigable boats, and so forth, with all references ever made in literature. It is a pity that he never gave us, as he could have done, the last word on Leonardo da Vinci as an engineer. It would, in fact, have been a word full of profound admiration.

The editor must have found difficulty in obtaining any photograph of Greenhill,—or one he ever possessed of any of his friends. The reason is not well known, but may now be given. He considered that no photograph was of any value, as a criterion of the personality of its subject, unless it was of life-size. His greatest friend at one time, and the recipient at this, and all later times of his unbridled admiration, was James Clerk Maxwell. The photograph of Maxwell's head and shoulders, magnified to life-size, crowned his mantelpiece in his quaint room in Staple Inn. It had few companions, but all had undergone the same treatment. The results, to many, certainly bore out his contention,—so much so that he never, if it could be prevented, allowed himself to be photographed, and actually destroyed other photographs of his friends if they would not accept this condition of their appearance in his gallery.

At one time he possessed all Maxwell's notebooks, in which Maxwell jotted down, in pencil, rough drafts of his scientific ideas, amidst a welter of domestic details, reminders of appointments, and so forth. He lent two once to a mutual friend, and, during a reconstruction of this friend's study, they were lost. Greenhill never forgave this, and in spite of the unfortunate man's fervent protestation that they contained only Maxwell's laundry bills, and no mention of electrical calculations, he was belaboured with Greenhill's umbrella in the Strand. The matter was ultimately adjusted. The fate of the five volumes Greenhill still had is unknown, but would be of great interest.

Many of Greenhill's private papers are now before me. To attempt, even in an anecdotal form, to give extracts, would lead at once to a whole volume. Perhaps no mathematician of this or any other generation had so many stories associated with him,—but all appreciative of his impulsive and lovable nature. Many show light on his old friends, especially Klein,—and his domestic troubles when he wished to work,—perhaps later somebody, after a sufficient lapse of years, may undertake the task of producing such a volume.

Greenhill loved conferences of any international kind; they did, in fact, constitute his holidays, and he followed them up for weeks later by visits to his friends. He officially disappeared at the end, and his whereabouts was

never known except to the friend he was then calling upon. At the British Association meeting in Canada in 1925, he created a scare even in our newspapers, for he was presumed lost before the meeting started. But he had only hurried on in front, and was found comfortably installed in the Students' Union. He did not attend scientific societies to any great extent, but was a stimulating presence when he did. He was President of the London Mathematical Society in 1890, and received from it the De Morgan Medal in 1902. This distinction he seemed to value more than any other he ever received. Was it because, as hinted already, he had more in common with Augustus de Morgan, either as mathematician or man, than anyone before or since ?—the main difference was, perhaps, that De Morgan married, and Greenhill did not.

But in later years, he felt somewhat out of touch with the newer trend of work, while still carrying on his own with extraordinary vigour at such an advanced age. For in a letter, now before me, regarding the London Mathematical Society, he says regretfully that the younger generation appears to be interested only in convergence, function-theory, inequalities, and so forth, and implies that he is at least too old to give up the mathematical views he has always held. But he is not hostile, and never could be, to any new development of value.

Readers of the *Gazette* will not need to be told that he was President of the Mathematical Association in 1913, and will equally need no reminder of his address on that occasion. Our science has lost a unique figure, which will soon become legendary. J. W. N.

Math. Note. 929. [V. 7].

RENATI FRANCISCI SLUSII

MESOLABUM

SEV

DVÆ MEDIÆ PROPORTIONALES

INTER EXTREMAS DATAS

PER CIRCVLVM ET ELLIPSIM

VEL HYPERBOLAM

INFINITIS MODIS EHIBITÆ

Leodij Eburonum

CIϽ IϽC LIX

(*Extract from Preface.*)

Problematis non novi nec incelebris effectionem tibi damus, Amice Lector, sed nobilis adeo & antiqui, ut consecrare audeat origines suas, & ad oraculum referre.

* * * * * * * *

Itaque forsitan actum, quod aiunt, agere videbor, dum post tot Clarissimorum Virorum conatus, ejusdem Problematis contemplationem rursus aggredior. Sed nihilominus aliquid superesse credidi, in quo non inutiliter exercerer, cùm primùm illius naturam pressius examinavi. Non quod ex eorum numero sim, qui rectâ & circulo illud construere inani labore contendunt: sed quod viderem, illos etiam qui vel organicæ rationis, vel sectionum Conicarum necessitatem agnovere, tam paucas nobis ejusdem demonstrationes hactenus ostendisse.

* * * * * * * *

Slusius rebukes the waste of time in attempting the Delian problem by straight line and circle. For *demonstrationes* in the last line the edition of nine years later has *effectiones*, and alters *ehibitae* in the title to *exhibitae*.

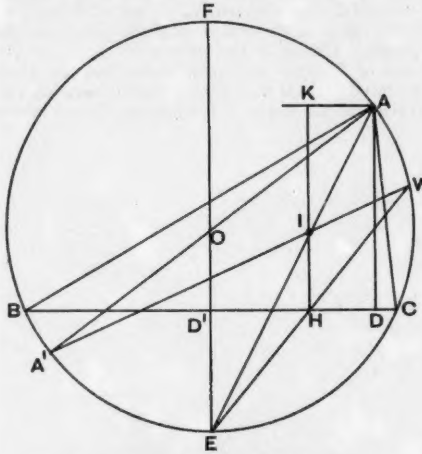
Prestwich Lodge, Cheltenham. A. A. Bourne.

MATHEMATICAL NOTES.

930. [K¹. 2. e.] *On a certain set of Twelve Circles all touching the Circumcircle.*

Theorem. If the altitudes of any triangle are drawn through the incentre, the escribed centres the twelve circles on these altitudes as diameters all touch the circumcircle.

Let I be the incentre of $\triangle ABC$ and let AI produced cut the circumcircle E; draw the diameters AOA', $ED'F$, the latter bisecting BC at right angles.



Draw AD perpendicular to BC and let KIH be the altitude equal and parallel to AD. Join EH and produce to cut the circumcircle in W.

Then $$EH \,.\, EW = ED' \,.\, EF = EC^2 = EI^2,$$

so $$E\hat{W}I = E\hat{I}H = E\hat{A}D = E\hat{A}A', \text{ since } AI \text{ bisects } O\hat{A}D;$$

$$\therefore\ A,\ I,\ W \text{ are collinear,}$$

so $$W \text{ lies on the circle on } AI \text{ as diameter;}$$

$$\therefore\ A\hat{W}K = A\hat{I}K = E\hat{I}H = I\hat{W}H;$$

$$\therefore\ H\hat{W}K \text{ is a right angle.}$$

Using E as centre and EI as radius of inversion the circumcircle inverts into BC and *vice versa*, while the circle on HK as diameter inverts into itself: and since this circle touches BC at H it must also touch the circumcircle at W. It is worth noting too that the inverse of the incircle touches the circumcircle at the same point W.

The rest of the theorem can be proved in a similar manner.

28th February, 1929. E. P. Lewis.

931. [X. 3.] *Example of formation of a Nomogram.*

A formula given for the velocity of water in a rectangular mill-stream whose breadth is x feet and depth y feet, is $\dfrac{500R}{0{\cdot}5+\sqrt{R}}$ feet per minute where $R=\dfrac{xy}{x+2y}$. A diagram is required from which to read off quickly the velocity

for different values of x and y, this velocity to be given in feet per minute and also in miles per hour. [Feet per minute is the more practically useful, as we can at ouce get vxy, the number of cubic feet flowing per minute, but miles per hour is also interesting.]

The equation for R in terms of x and y can be written $\frac{2R}{x}+\frac{R}{y}=1$; therefore if values of x and y are plotted along a pair of rectangular axes, the point $(2R, R)$ will lie on the line whose intercepts on the axes are the given, or required, values of x and y. This point also lies on the line through the origin drawn with gradient 1/2. Consequently, values of R can be labelled along this line by numbers equal to the ordinates of the points so labelled, as shown in the annexed graph. This gives the scale of R.

To find the value of R corresponding to given breadth (x) and depth (y) of the stream, all we have to do is to stretch a thread between the correct points on the axes, and read off the value of R when the thread crosses the R-scale.

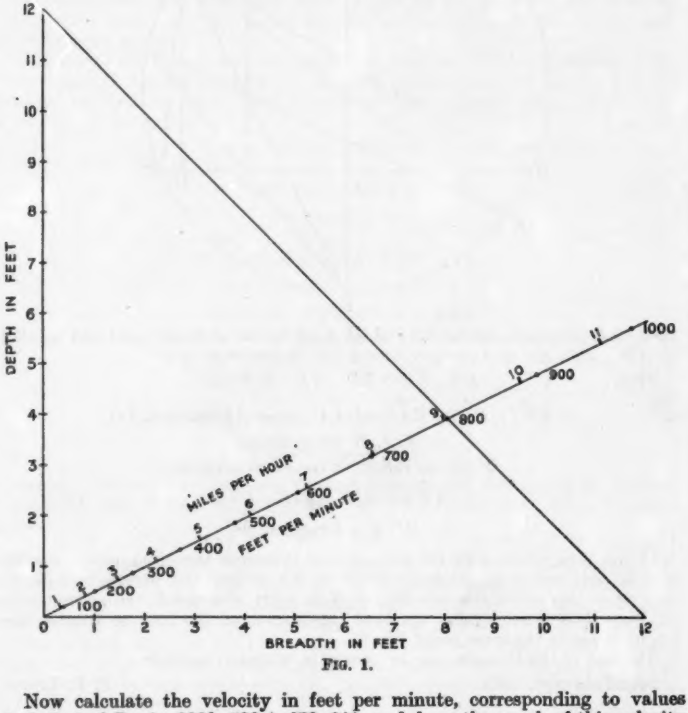


FIG. 1.

Now calculate the velocity in feet per minute, corresponding to values 1, 2, 3, 4 of R, viz. $333\frac{1}{3}$, 522·4, 672, 800, and draw the graph of this velocity as a function of R, as shown.

This scale of velocities is shown by an upright line on the right; and the scale of miles per hour is easily deduced from it by drawing a sloping line from its foot to a point $9\frac{1}{11}$ divisions to the right of the 800-feet-per-minute point, since 800 feet per minute $=9\frac{1}{11}$ miles per hour. [1 division $=\frac{1}{8}$ inch.]

A JOHNIAN TRINITY, 1870.

R. PENDLEBURY.

E. L. LEVETT. A. G. GREENHILL.

St. John's secured the first three places in the Tripos of that year.

This completes the graph, which is a combination graph and nomogram.

In the diagram is the solution of the question what depth the stream should be if the breadth was 4 feet and the speed had to be 5 miles per hour. The answer is $6\frac{1}{2}$ feet, as shown by the threadline ADC.

From the above diagram a simple nomogram can be formed, ignoring any explicit reference to R, by plotting the velocity scale directly on to the R line. This is done in the following figure which is the complete nomogram (on a larger scale) from which to read off the velocity either in feet per minute or in miles per hour, by just laying a straight-edge between two desired points and reading off the third quantity. The nomogram consists merely of three scaled lines, two being uniform, and the third scaled according to the graph which is now no longer needed.

The cross-line drawn from $x=12$ to $y=12$ shows that with this breadth and depth the velocity would be 800 feet per minute, or just over 9 miles per hour. If we wish to know the corresponding value of R it can be read off

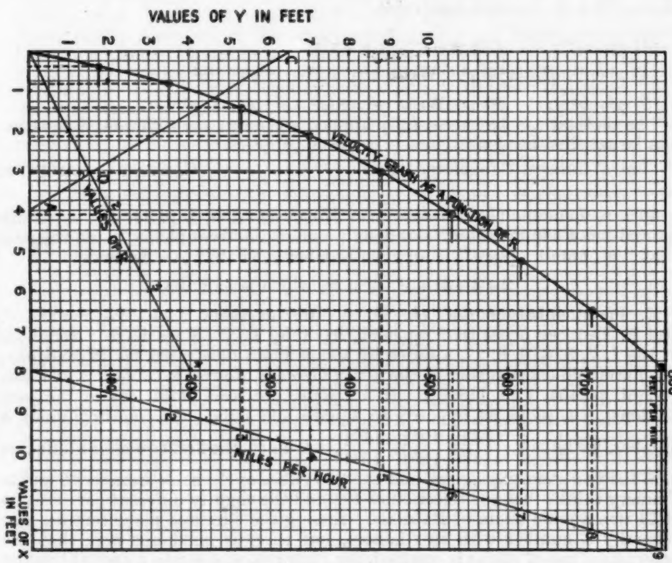


FIG. 2.

Nomogram of velocity of rectangular mill-stream.

the y scale on the same level as the crossing point (in this case, 4). But R is not needed in using the diagram, so need not be alluded to except when explaining the constructions. ALFRED LODGE.

932. [H. 2. a.] *The Method of Parameters in solving certain types of Partial Differential Equations of the First Order.*

The method indicated in the following examples, although theoretically not new, deserves to be more widely known and applied than is apparent from text-books on Differential Equations. It will be seen that the method includes

completely the Standard Forms I and III (Forsyth's *Treatise on Differential Equations*, chap. ix), and is more general, since many examples which have to be reduced to one of these standard forms by a suitable transformation of variables can be worked straight off. Certain particular examples coming under Standard II can also be worked out easily. I give the name "The Method of Parameters" after its analogy with the method of Analytical Geometry.

Examples :

(1) To solve $\frac{p^2}{x^2}+\frac{q^2}{y^2}=1$. Put $p=x\cos\alpha$; $q=y\sin\alpha$. Then

$$dz=x\,dx\cos\alpha+y\,dy\sin\alpha$$

leads to the complete integral.

(2) $p^2x^2+q^2y^2=z$. Put $p=\frac{\sqrt{z}}{x}\cos\alpha$; $q=\frac{\sqrt{z}}{y}\sin\alpha$. With these values, $dz=pdx+q\,dy$ becomes integrable.

(3) $pq=x^my^nz^l$. Put $p=x^mz^{\frac{l}{2}}$; $q=y^nz^{\frac{l}{2}}$.

(4) $z^2(p^2+q^2)=x^2+y^2$. Write $z^2p^2=x^2+c$; $z^2q^2=y^2-c$.

(5) $p^2+q^2=(x^2+y^2)z$. Write $p^2=z(x^2+c)$; $q^2=z(y^2-c)$.

The method can be successfully applied whenever the differential equation can be satisfied by $p=\phi_1(x)\psi(z)$; $q=\phi_2(y)\psi(z)$.

Central College, Bangalore, India. C. N. SRINIVASIENGAR.

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667. E. Lemoine (*Intermédiaire des Mathématiciens*, I. p. 83) in the passage quoted has a comma after "des deux premiers nombres pleins", and suggests that "un nombre plein" is an even number. In that case he finds

$$2+4+4^2+4^2+2^3+2^3, \text{ which is half } 108.$$

E. Fauquembergue (*loc. cit.* II. p. 102) suggests $2+4+2^2+3^2+2^3+3^3$, and P. Tannery notes that replacing $2+4$ by $1+2+3$ we have a reminder of the 1, 2, 3, 4, 9, 8, 27, a series by which Plato illustrated his Psychogony, the harmonic constitution of the soul (*Timaeus*, Jowett's *Dialogues of Plato*, vol. iii. p. 454, § 36, [1892]). If for *pleins* we read *plains* (as in the best editions), *i.e.* *plans* as opposed to square and cube, not counting unity a number, we have 1, $\begin{Bmatrix}2\\3\end{Bmatrix}$, $\begin{Bmatrix}2^2\\3^2\end{Bmatrix}$, $\begin{Bmatrix}2^3\\3^3\end{Bmatrix}$, *i.e.* unity, the first two *plains* numbers 2, 3, the two *quadrangulaires*, and the two cubes of the same numbers 2 and 3. Tannery thinks that the selection of 108 and the slightly different composition of 54 was a mere whim of Rabelais. Among the Greeks a plane number was generally a number with two factors, but there is no doubt here that the exceptional use of *plains* is justified, for Plutarch, who wrote a special treatise Περὶ τῆς ἐν Τιμαίῳ ψυχογονίας, uses elsewhere (*De animae procreatione in Timaeo*, c. xi.) πρώτους ἐπιπέδους for 2 and 3.

668. AN ANAGRAM.—The practice of enclosing discoveries in sealed packets and sending them to academies, seems so inferior to the old one of Huyghens, that the following is sent you for publication in the old conservated form :

$A^8C^3DE^{12}F^4GH^6I^6L^3M^2N^5O^6PR^4S^5T^{14}U^6V^2WXY^2$ WEST.

[Stewart and Tait's *Unseen Universe* was about to appear. *Nature*, vol. x. p. 481. Oct., 5, 1874. The sentence embalmed in the anagram is: "Thought conceived to affect the matter of another universe simultaneously with this may explain a future state."

The signature is to be interpreted—We S(tewart) T(ait).]

REVIEWS.

Collected Papers of Srinivasa Ramanujan. Edited by G. H. Hardy, P. V. Seshu Aigar, and B. M. Wilson. Pp. xxxvi + 355. 30s. net. 1927. (Cam. Univ. Press.)

Ramanujan was born in India in December 1887, came to Trinity College, Cambridge, in April 1914, was ill from May 1917 onwards, returned to India in February 1919, and died in April 1920. He was a Fellow of Trinity and a Fellow of the Royal Society.

Ramanujan had no university education, and worked unaided in India until he was twenty-seven. When he was sixteen he came by chance on a copy of Carr's *Synopsis of Mathematics*; and this book, now sure of an immortality its author can hardly have dreamt of, woke him quite suddenly to full activity. A study of its contents is indispensable to any considered judgment. It gives a very full account of the purely formal side of the integral calculus, containing, for example, Parseval's formula, Fourier's repeated integral and other "inversion formulae", and a number of formulae of the type recognizable by the expert under the general description "$f(\alpha)=f(\beta)$ if $\alpha\beta=\pi^2$". There is also a section on the transformation of power series into continued fractions. Ramanujan somehow acquired also an effectively complete knowledge of the formal side of the theory of elliptic functions (not in Carr). The matter is obscure, but this, together with what is to be found in, say, Chrystal's *Algebra*, seems to have been his complete equipment in analysis and theory of numbers. It is at least certain that he knew nothing of existing methods of working with divergent series, nothing of quadratic residuacity, nothing of work on the distribution of primes (he may have known Euler's formula $\Pi(1-p^{-s})^{-1}=\Sigma n^{-s}$, but not any account of the ζ-function). Above all, he was totally ignorant of Cauchy's theorem and complex function-theory. (This may seem difficult to reconcile with his complete knowledge of elliptic functions. A sufficient, and I think a necessary, explanation would be that Greenhill's very odd and individual *Elliptic Functions* was his text-book.)

The work he published during his Indian period did not represent his best ideas, which he was probably unable to expound to the satisfaction of editors. At the beginning of 1914, however, a letter from Ramanujan to Mr. Hardy (then at Trinity, Cambridge) gave unmistakable evidence of his powers, and he was brought to Trinity, where he had three years of health and activity. (Some characteristic work, however, belongs to his two years of illness.)

I do not intend to discuss here in detail the work for which Ramanujan was solely responsible (a very interesting estimate is given by Prof. Hardy, p. xxxiv). If we leave out of account for the moment a famous paper written in collaboration with Hardy, his definite contributions to mathematics, substantial and original as they are, must, I think, take second place in general interest to the romance of his life and mathematical career, his unusual psychology, and above all to the fascinating problem of how great a mathematician he might have become in more fortunate circumstances. In saying this, of course, I am adopting the highest possible standard, but no other is appropriate.

Ramanujan's great gift is a "formal" one; he dealt in "formulae". To be quite clear what is meant, I give two examples (the second is at random, the first is one of supreme beauty):

$$p(4)+p(9)x+p(14)x^2+\ldots=5\frac{\{(1-x^5)(1-x^{10})(1-x^{15})\ldots\}^5}{\{(1-x)(1-x^2)(1-x^3)\ldots\}^6},$$

where $p(n)$ is the number of partitions of n;

$$\int_0^\infty \frac{\cos \pi x}{\{\Gamma(\alpha+x)\Gamma(\alpha-x)\}^2}\,dx=\frac{1}{4\Gamma(2\alpha-1)\{\Gamma(\alpha)\}^2}\;(\alpha>\tfrac{1}{2}).$$

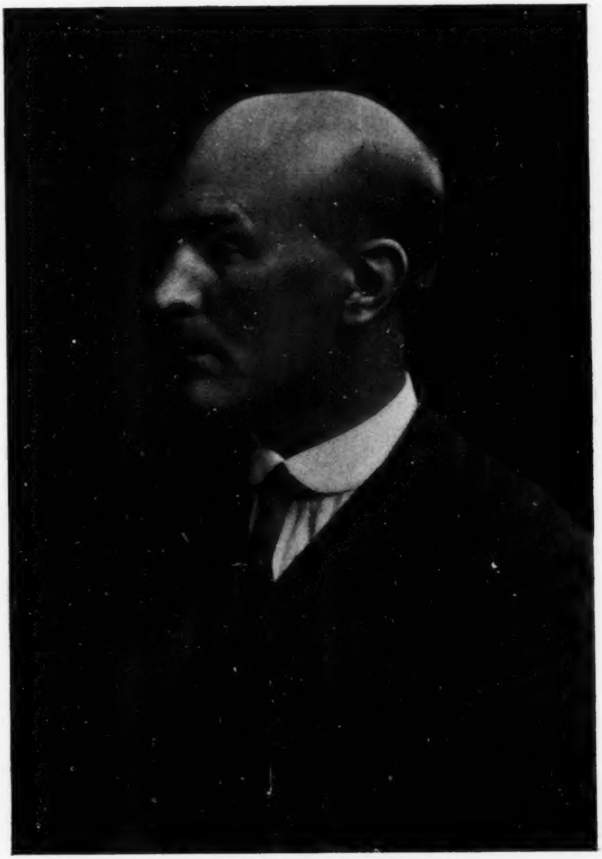
But the great day of formulae seems to be over. No one, if we are again to take the highest standpoint, seems able to discover a radically new type, though Ramanujan comes near it in his work on partition series; it is futile to multiply examples in the spheres of Cauchy's theorem and

elliptic function theory, and some general theory dominates, if in a less degree, every other field. A hundred years or so ago his powers would have had ample scope. Discoveries alter the general mathematical atmosphere and have very remote effects, and we are not prone to attach great weight to rediscoveries, however independent they seem. How much are we to allow for this; how great a mathematician might Ramanujan have been 100 or 150 years ago; what would have happened if he had come into touch with Euler at the right moment? How much does lack of education matter? Was it formulae or nothing, or did he develop in the direction he did only because of Carr's book—after all, he learned later to do new things well, and at an age mature for an Indian? Such are the questions Ramanujan raises; and everyone has now the material to judge them. The letters and the lists of results announced without proof are the most valuable evidence available in the present volume; they suggest, indeed, that the note-books would give an even more definite picture of the essential Ramanujan, and it is very much to be hoped that the editors' project of publishing them *in extenso* will eventually be carried out.

Carr's book quite plainly gave Ramanujan both a general direction and the germs of many of his most elaborate developments. But even with these partly derivative results one is impressed by his extraordinary profusion, variety, and power. There is hardly a field of formulae, except that of classical number-theory, that he has not enriched, and in which he has not revealed unsuspected possibilities. The beauty and singularity of his results is entirely uncanny. Are they odder than one would expect things selected for oddity to be? The moral seems to be that we never expect enough; the reader at any rate experiences perpetual shocks of delighted surprise. And if he will sit down to an unproved result taken at random, he will find, if he can prove it at all, that there is at lowest some "point", some odd or unexpected twist. Prof. Watson and Mr. Preece have begun the heroic task of working through the unproved statements; some of their solutions have appeared recently in the *Journal of the London Mathematical Society*, and these strongly encourage the opinion that a complete analysis of the note-books will prove very well worth while.

There can, however, be little doubt that the results showing the most striking originality and the deepest insight are those on the distribution of primes (see pp. xxii-xxv, xxvii, 351, 352). The problems here are not in origin formal at all; they concern approximative formulae for such things as the number of primes, or of integers expressible as a sum of two squares, less than a large number x; and the determination of the orders of the errors is a major part of the theory. The subject has a subtle function-theory side; it was inevitable that Ramanujan should fail here, and that his methods should lead him astray; he predicts the approximative formulae, but is quite wrong about the orders of the errors. These problems tax the last resources of analysis, took over a hundred years to solve, and were not solved at all before 1890; Ramanujan could not possibly have achieved complete success. What he did was to perceive that an attack on the problems could at least be begun on the formal side, and to reach a point at which the main results became plausible. The formulae do not in the least lie on the surface, and his achievement, taken as a whole, is most extraordinary.

If Carr's book gave him direction, it had at least nothing to do with his *methods*, the most important of which were completely original. His intuition worked in analogies, sometimes remote, and to an astonishing extent by empirical induction from particular numerical cases. Being without Cauchy's theorem, he naturally dealt much in transformations and inversions of order of double integrals. But his most important weapon seems to have been a highly elaborate technique of transformation by means of divergent series and integrals. (Though methods of this kind are of course known, it seems certain that his discovery was quite independent.) He had no strict logical justification for his operations. He was not interested in rigour, which for that matter is not of first-rate importance in analysis beyond the undergraduate state, and can be supplied, given a real idea, by any competent professional. The clear-cut idea of what is *meant* by a proof, nowadays so familiar as to be taken for



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A. W. SIDDONS.

granted, he perhaps did not possess at all. If a significant piece of reasoning occurred somewhere, and the total mixture of evidence and intuition gave him certainty, he looked no further. It is a minor indication of his quality that he can never have *missed* Cauchy's theorem. With it he could have arrived more rapidly and conveniently at certain of his results, but his own methods enabled him to survey the field with an equal comprehensiveness and as sure a grasp.

I must say something finally of the paper on partitions (pp. 276-309) written jointly with Hardy. The number $p(n)$ of the partitions of n increases rapidly with n, thus:

$$p(200)=3972999029388.$$

The authors show that $p(n)$ is the integer nearest

$$(1) \qquad \frac{1}{2\sqrt{2}}\sum_{q=1}^{\nu} \sqrt{q}A_q(n)\psi_q(n),$$

where $A_q(n)=\Sigma\omega_{p,q}e^{-2np\pi i/q}$, the sum being over p's prime to q and less than it, $\omega_{p,q}$ is a certain $24q$th root of unity, ν is of the order of $\sqrt{n}$, and

$$\psi_q(n)=\frac{d}{dn}(\exp\{C\sqrt{(n-\tfrac{1}{24})}/q\}), \quad C=\pi\sqrt{\tfrac{2}{3}}.$$

We may take $\nu=4$ when $n=100$. For $n=200$ we may take $\nu=5$; five terms of the series (1) predict the correct value of $p(200)$. We may always take $\nu=\alpha\sqrt{n}$ (or rather its integral part), where α is any positive constant we please, provided n exceeds a value $n_0(\alpha)$ depending only on α.

The reader does not need to be told that this is a very astonishing theorem, and he will readily believe that the methods by which it was established involve a new and important principle, which has been found very fruitful in other fields. The story of the theorem is a romantic one. (To do it justice I must infringe a little the rules about collaboration. I therefore add that Prof. Hardy confirms and permits my statements of bare fact.) One of Ramanujan's Indian conjectures was that the first term of (1) was a very good approximation to $p(n)$; this was established without great difficulty. At this stage the $n-\frac{1}{24}$ was represented by a plain n—the distinction is irrelevant. From this point the real attack begins. The next step in development, not a very great one, was to treat (1) as an "asymptotic" series, of which a fixed number of terms (*e.g.* $\nu=4$) were to be taken, the error being of the order of the next term. But from now to the very end Ramanujan always insisted that much more was true than had been established: "there must be a formula with error $O(1)$." This was his most important contribution; it was both absolutely essential and most extraordinary. A severe numerical test was now made, which elicited the astonishing facts about $p(100)$ and $p(200)$. Then ν was made a function of n; this *was* a very great step, and involved new and deep function-theory methods that Ramanujan obviously could not have discovered by himself. The complete theorem thus emerged. But the solution of the final difficulty was probably impossible without one more contribution from Ramanujan, this time a perfectly characteristic one. As if its analytical difficulties were not enough, the theorem was entrenched also behind almost impregnable defences of a purely formal kind. The form of the function $\psi_q(n)$ is a kind of indivisible unit; among many asymptotically equivalent forms it is essential to select exactly the right one. Unless this is done at the outset, and the $-\frac{1}{24}$ (to say nothing of the $\frac{d}{dn}$) is an extraordinary stroke of formal genius, the complete result can never come into the picture at all. There is, indeed, a touch of real mystery. If only we *knew* there was a formula with error $O(1)$, we might be forced, by slow stages, to the correct form of ψ_q. But why was Ramanujan so certain there *was* one? *Theoretical* insight, to be the explanation, had to be of an order hardly to be credited. Yet it is hard to see what numerical instances could have been available to suggest so strong a result. And unless the form of ψ_q was known already, *no* numerical evidence could suggest anything of the kind—there seems no escape, at least, from the conclusion that the discovery of the correct form was a single stroke of insight. We owe the theorem to a singularly

happy collaboration of two men, of quite unlike gifts, in which each contributed the best, most characteristic, and most fortunate work that was in him. Ramanujan's genius did have this one opportunity worthy of it.

The volume contains a biography by the second of the editors, and the obituary notice by Prof. Hardy. These give quite a vivid picture of Ramanujan's interesting and attractive personality. The mathematical editors have done their work most admirably. It is very unobtrusive; the reader is told what he wants to know at exactly the right moment, and more thought and bibliographical research must have gone into it than he is likely to suspect.

J. E. LITTLEWOOD.

A Course of Pure Mathematics. By G. H. HARDY, M.A., D.Sc., LL.D., F.R.S., Fellow of New College, Savilian Professor of Geometry in the University of Oxford, late Fellow of Trinity College, Cambridge. Pp. xii + 456. 12s. 6d. net. Fifth edition. 1928.

I am very glad to welcome a fifth edition of Prof. Hardy's book, not so much for the sake of the changes and corrections made, as for the evidence afforded by its publication that so admirable a book is still being sold (and presumably read) on a considerable scale.

The most important change is the addition of an appendix on Landau's symbols O, o, $\sim$, which are now in common use in higher analysis, and may profitably be introduced at a fairly early stage to simplify and abbreviate the discussion of problems of limits and approximations when what we want to prove is (loosely stated) that certain "remainders" are negligible as compared with what we retain.

It is an interesting illustration of the difficulty of ensuring accuracy in detail, however accurate the mind of the writer may be, that a sentence in the preface, calling attention to a correction in the text, itself requires correction, as the reference should be to § 9, not to p. 9. I notice also that of three similar examples (App. I. Exx. 1, 6, 7) which caught my eye in the first edition as (trivially) incorrect, owing to an ambiguity of sign, only Ex. 6 has been corrected.

An interval of just twenty years separates the present from the first edition. Though the general plan of the book has not been substantially changed, it has become perceptibly harder. Dedekind's theory of real numbers, the Heine-Borel theorem, uniform continuity, upper and lower "bounds", the elements of the theory of implicit functions, and other topics have replaced some comparatively commonplace matter belonging to analytical geometry and trigonometry, most of the changes having taken place between the first and second editions. The changes correspond not unfairly, though perhaps with some exaggeration, to the progress in the analytical knowledge and skill of the competent young mathematician which has taken place in the last twenty years, much of which is undoubtedly due to Prof. Hardy's book and to the teaching of younger analysts who have been taught or influenced by him. It is refreshing to compare the grasp of fundamental analytical ideas of the better mathematical students whom I now meet with the slipshod notions and methods of my undergraduate contemporaries and of my earlier pupils.

On the other hand, as the capacity of the human mind is probably fairly constant and the time available for mathematical study does not appear to have increased appreciably, there has been almost inevitably some loss to correspond with this important advance; and I seem to notice an appreciable diminution in the power of manipulation and in knowledge of straightforward parts of analysis which the average mathematician has to use in his ordinary work. This is, however, in part due to the lack of a satisfactory book, at any rate in English, dealing with parts of the differential and integral calculus which lie outside of the scope of Prof. Hardy's book. When a student inquires where he can read about, say, Jacobians, or maxima and minima of two variables, or the evaluation of "definite integrals" by real methods, or double integrals, to mention only a few interesting and important topics, I have to refer him either to an old-fashioned text-book, sadly lacking in rigour but full of information, or to a French book which he may be unable to read, or (perhaps more frequently) may imagine that he cannot read.

There seems to be an urgent need for a substantial treatise on analysis, covering the ground or part of the ground between the content of Prof. Hardy's book and of much more advanced and really formidable treatises such as Prof. Hobson's *Functions of a Real Variable* and the *Modern Analysis* by Profs. Whittaker and Watson. It has been a tradition, much to the advantage of students, that eminent French mathematicians should publish books of approximately this character, based in general on their university lectures. I recall the names of Moigno, Serret, Bertrand, Jordan, Humbert, Picard, Goursat, Vallée Poussin (though I am aware that the last name should not occur in a list of Frenchmen), as authors of *Cours* or *Traités d'Analyse*, written with the competence of scholars and the lucidity which by no means every scholar can command. Is it too much to hope that Prof. Hardy will sometime write a book on some such lines, with, of course, the changes required to suit English conditions ? The work could easily be lightened by omitting the differential equations, which traditionally form part of a French *Cours d'Analyse*, but could quite conveniently be the subject of a separate treatise. I can think of no one who could do it so well, as, in addition to learning and lucidity at least equal to that of the French school, he has shown in this book and elsewhere a power of being interesting, which is to my mind unequalled by any of the eminent men (with the possible exception of M. Picard) whom I have just mentioned. I suggest to Prof. Hardy that he would probably be increasing his service to English mathematics if he were to divert to this purpose some of the mental energy and time that he would otherwise devote to drawing somewhat closer the *cordon* that surrounds the unknown zeroes of Riemann's Zeta-function, and to similar problems.

King's College, Cambridge, April 1929. ARTHUR BERRY.

(1) **Sir Isaac Newton.** By C. D. BROAD. Annual Lectures on a Master Mind: Henrietta Hertz Trust of the British Academy. Pp. 32. 2s. net. 1927. (Milford, Ox. Univ. Press.)

(2) **Sir Isaac Newton, 1727-1927: A Bicentenary Evaluation of his Work.** A Series of Papers prepared under the Auspices of The History of Science Society, in collaboration with the American Astronomical Society, the American Mathematical Society, the American Physical Society, the Mathematical Association of America, and various other Organisations. Pp. ix + 351. 22s. 6d. net. 1928. (London : Baillière, Tindall and Cox.)

(1) "Let us now praise famous men" is a call that is inevitable on the occasion of centenaries. Certain names always live as representing a great epoch, and the call is irresistible when the subject of commemoration is an ineffaceable landmark in the intellectual history of mankind.

It was natural that a lecture to the British Academy on "a master mind" should in 1927 be devoted to Newton. Within the limited compass of the time that human endurance extends to an address, Dr. Broad gave an admirable survey of Newton's life and creative activity. Incidentally he mentioned that he occupies Newton's old rooms in Trinity on "that distinguished staircase," E. Great Court, a fact suggesting to him the perhaps unduly pessimistic lines :

Aetas parentum, peior avis, tulit
Nos nequiores, mox daturos
Progeniem vitiosiorem.

Before passing on, we cannot refrain from quoting one or two of the lecturer's happy touches :

The theory of fluxions "reached its high-water mark of logical rigour in the profound but unreadable *Treatise* of Maclaurin. The fluxional method achieved purity at the expense of fertility. 'Like a virgin dedicated to God,' to quote Bacon, 'it produced nothing.'"

Leibniz "unblushingly used infinitesimals, and neglected their squares and higher powers with a gay indifference which recalls Rousseau's treatment of his illegitimate children."

"Newton was a convinced supporter of that great party which counts St. Thomas Aquinas and Mr. Locke as its political philosophers, and the Devil as its first member."

And of the re-coinage—"if Newton under Jeffreys had been called upon to withstand the *vox instantis tyranni*, he and Montague had now to meet the full blast of *civium ardor prava iubentium*."

(2) On 25th and 26th Nov. 1927, the bicentenary of Newton's death was celebrated in New York by an exhibition of "Newtoniana and Related Material," and by the delivery of the series of addresses now published in the handsome volume before us. The publication of the addresses was arranged by the History of Science Society. The President of that body, Prof. David Eugene Smith, *doyen* of American historians, contributes the Introduction and a short address entitled "Newton in the Light of Modern Criticism."

Whether the occasion was appropriate for a recapitulation of the charges made against Newton's character and motives is a matter of opinion. We incline to Bolingbroke's attitude when he wrote to Marlborough: "he was so great a man that I forgot he had that defect." But it has been said of the Muse of History that when she pauses and dwells upon the meanness of a Bacon, or the drivellings of a Swift, it is merely a literary artifice. She hopes thereby to heighten the tones of her picture, to give additional sparkle to the tears she will presently let fall over human frailty. After all, what matters here and now is Prof. Smith's conclusion. He briefly recounts the accusations and closes as follows:

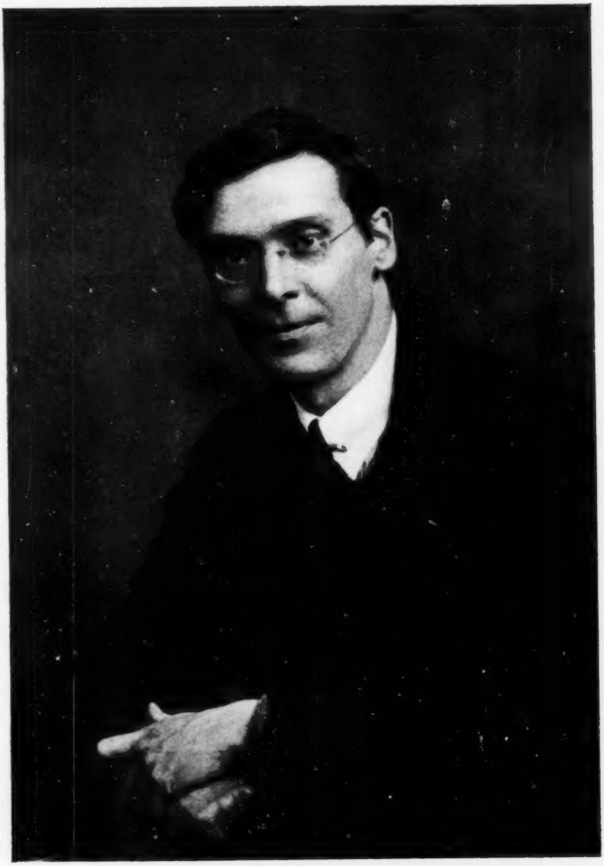
"When we review his life, his idiosyncracies, his periods of contrast, and his doubts and ambitions and desire for place, may we not take some pleasure in thinking of him as a man—a man like most other men save in one particular—he had genius—a greater touch of divinity than comes to the rest of us? Few men have ever lived who explored so successfully as wide a range of human activities and few who could so justly have used the well-known phrase, *Homo sum, et nihil humani a me alienum puto.*"

Prof. D. C. Miller deals with the debt of Optics to Newton, summarises his contributions, specifies discoveries fathered upon him for which he was not responsible—*e.g.* Newton's Rings—and points out several phenomena which it is strange that he did not detect. As to the ultimate fate of the Newtonian theory of light, he concludes: "Whether a corpuscular theory, an ether theory, either one or both combined, will prevail when a Newton tercentenary is celebrated, one would not now venture to assert. The temptation to draw analogies between Newton's corpuscles and Planck's quanta, and between fits of easy transmission and waves, is confronted by difficulties so far insuperable, unless the new mechanics of Heisenberg & Schrödinger provides the necessary reconciliation." It is interesting to compare this statement with the closing remarks of Sir Richard Glazebrook in the Supplement to *Nature*, 26th March, 1927, p. 48.

"Newton's Philosophy of Gravitation with special reference to Modern Relativity Ideas" is the subject of Prof. G. D. Birkhoff's clear and succinct discourse. "As a first approximation, the Newtonian dynamics with its spatial relativity is likely to stand permanently. It is the simplest theory which explains the main facts. The gravitational theory which is the corner-stone of his dynamics, will stand for the same reason." And again, "any more elaborate theory would have been a useless and unjustified flight of the imagination." But as the years passed on experimental results in physics began to accumulate and our views of the physical universe had to be altered. Einstein launched his gravitational theory of relativity in 1915, and we are now at "the stage in which no theories appear to be fundamental in physics—it is merely that some are more fundamental than others in certain directions."

In a short sketch entitled "Newton's Dynamics," Prof. M. I. Pupin shows how, with the arrival on the scene of electrical science, the Newtonian concepts of mass and momentum acquire a deeper meaning, perhaps "expressible in terms of the Newtonian actions and reactions of the electron and its field. This belief inspires the hope that a new dynamical science will soon be born, and that, like Maxwell's dynamics, it will be another daughter of Newton's dynamics."

Dr. P. R. Heyl deals with "Newton as an Experimental Philosopher," his "partly innate" skill with his hands, his facilities for work, and his experimental ingenuity. "When we think of Newton as Noyes calls him, 'the king of thought,' let us also bear in mind that upon which his thought was founded and without which thought is as barren as faith without works—painstaking, careful experiment."



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PROF. E. H. NEVILLE.

We next come to a valuable contribution by Prof. W. W. Campbell. The Director of the Lick Observatory takes as his subject: "Newton's Influence upon the Development of Astrophysics." He shows how "as an instrument of prediction and deduction; as an agency of interpretation; and as a check upon hypotheses" the law of gravitation is invaluable to the astrophysicist. From "a long list of available cases" he selects as illustrations the internal conditions of the stars, the orbits of the comets, the source of light in the distant stars, the chemical constitution of the heavenly bodies, and so on. Astronomers have realised in certain fields the exceptional value of reflecting telescopes, the first of which was constructed by Newton in 1665 after recalling the theoretical descriptions given by Père Mersenne in 1639 and by James Gregory in 1662. Even his views as to the nature of light have "much in common with those held by physicists in the last two or three years—by de Broglie, Schrödinger, and others." And the lecture ends with the striking statement: "To me it is clear that Sir Isaac Newton, easily the greatest man of physical science in historic times, was uniquely the great pioneer of astrophysics."

From Prof. E. W. Brown we have an essay on "Developments following from Newton's work." He begins with a reference to the necessity of research in the pure laws of mechanics—a need that in no far distant day will be realised by manufacturers of moving machinery. In other words, the mere knowledge of the laws of motion is not enough for the engineer. Lack of care and lack of knowledge in their application lead to failure to predict what must happen when machines are moving at high speeds. It is the power to predict that is "the chief service which Newton rendered to mankind." He then illustrates his case from his own subject, aptly comparing the contest between observer and mathematician to that between the makers of guns and armour plate. Theory caught up with observation in due course. "Leverrier for the planets and Hansen for the moon appeared to have completed the task begun by Newton two centuries earlier." And so the lecturer brought his audience to the verdict: "Newton's name runs no danger of being forgotten when his work is subjected to the acid test for all scientific work, namely, its capacity for further development."

Prof. Florian Cajori adds to his reputation by his exhaustive treatment of the famous question of "Newton's Twenty Years' Delay in announcing the Law of Gravitation." He takes up over sixty pages of the volume, and not a page of them is without interest. He first discusses the various estimates of the size of the earth that were current in England before Picard's measurements, and the value of the degree generally accepted by navigators and geographers; he carefully tabulates the results, and concludes that the delay could not have been due to any serious uncertainty as to the value assumed in the calculations of 1666. He re-examines the suggestion connected with the names of Adams and Glaisher, that the delay was due to theoretical difficulties in the problem of the attraction of a sphere—difficulties not finally removed by Newton until 1685. Once Newton solved this problem, the rest was easy. "It put the law of gravity," says Prof. H. H. Turner, "on an entirely new footing, converting what had been possibly only a crude approximation for planetary bodies into an accurate and universal law. The discovery of it was a great achievement, representing an important step in integral calculus, and it is probable that the final success in solving it was only the sequel to many failures." It was these "many failures" that were the cause of the delay. Prof. Cajori also contributes a short paper on another moot question, deciding in favour of the claim that as early as 1671, Newton used the fundamental processes of partial differentiation and partial integration.

We have no space to dwell on Prof. L. C. Newell's address on Newton's work in alchemy and chemistry, much of the contents of which will be no surprise to those who possess our own commemorative Newton number.

Newton's place in the history of religious thought is so rarely alluded to in these days that it seems to deserve a reference. It is discussed in this volume by Prof. G. S. Brett. In that history, the life and work of Newton have a double significance. "The special significance belongs to his theo-

logical writings: the general significance is discovered in that natural philosophy which comprised both his theory of the universe and his idea of its divine government." Men in those days had to remember the limitations on freedom imposed by the Toleration Act of 1689. Whiston, Newton's successor in the Lucasian chair, was expelled from the university and lost his professorship (1711) for his outspoken Arianism. What induced Newton to tread as dangerous a path when he brought "the higher criticism" of his time to bear upon 1 John v. 7* and 1 Tim. iii. 16? Prof Brett is inclined to think that it was not so much an interest in the doctrine of the Three Persons as "a love of mathematical accuracy." A similar suggestion covers the attention paid by Newton to the prophecies of Daniel and the Apocalypse of St. John. The fact of prophecy and the significance of the apocalyptic language were accepted as axioms. But he was specially attracted by the "astronomical symbolism" of "the apocalyptic writings": "the books seemed to him examples of a method as symbolic as the mathematical." Prof. Brett refrains from conjecture as to the specific value that can be attached to Newton's conclusions. He next notes the suggestion that Newton embarked on these speculations at a time when he was suffering from mental instability. Certainly Bishop South, who had some experience, laid down in his homely way that: "if the Apocalypse does not find a man mad, it leaves him so." But as a matter of fact, Newton, like most men of culture of the time, in the universities and elsewhere, had been interested in such questions long before the death of his mother (in 1690), an event which is supposed to have caused his mental disturbance and turned his mind to theology.

Prof. Cajori has kindly sent me a reprint of a paper contributed by him to *Popular Astronomy*, vol. xxxiv. No. 2, Feb. 1926. He has had the opportunity of examining a volume by Henry More, the Cambridge Platonist, on the Prophet Daniel and the Apocalypse (1681). The flyleaf bears the inscription in Newton's handwriting: *Is. Newton. Ex dono Reverendi Authoris.* The pages of the text are rich in marginal annotations in the same hand. We may take it, suggests the writer of the paper, that the years of Newton's great creative effort were 1685-1687, when the MSS. of the *Principia* were in hand. It is therefore extremely probable that the annotations were written, perhaps on the receipt of the presentation copy from his friend, or at any rate between 1681 and 1685. This is a fairly reasonable supposition, especially if coupled with inferences from the handwriting of Newton—in 1682, for instance—and with the fact that More's name is not mentioned in Newton's writings on theology printed in his later years.

Further, it would seem to dispose of the ascription "of all Newton's theological interests to a temporary loss of mental power," which found favour in many quarters, and with which suggestions Prof. Brett will have nothing to do. Nor will Dr. Broad: "If any one be left so steeped in the scientific orthodoxy of the eighteen-seventies as to consider an interest in theology or alchemy to be a clear sign of mental decay, he may be reminded that Newton was also engaged at this time in working out . . . one of the hardest parts of gravitational astronomy." Enormous interest was, of course, taken in any views supposed to be held by so eminent a man in subjects considered to be necessarily outside his own particular sphere of investigation. Many, indeed, held quite frankly that he was an infidel. What was really at the back of his mind is well put in what Colin Maclaurin heard him say, "that it gave him particular pleasure that his philosophy had promoted attention to final causes," and the passage adds: "his followers who have not rivalled his genius have not degenerated from his piety." There were others, such as the poet Thomson, who thought of him (a couple of months after his death) as now "the beloved of Heaven," . . .

> ". . . now he wanders through those endless worlds
> He here so well descried, and wondering talks,
> And hymns their Author with his glad compeers."

What Whiston thought is well known, but what was confided to a correspondent by William Warburton reflects the truculence allowed to themselves

* Prof. Brett gives 1 John ii. 7.

by critics in those days. This versatile but intolerant prelate—"the stupidest boy that was ever in Newark School," said his old headmaster—and as Coleridge deemed him, "the thought-swarming but idea-less Warburton," wrote as follows on Newton's chronological and theological publications: "A word in your ear—what Sir Isaac wrote of the Egyptian antiquities is the most wretched thing ever wrote by any body. But more of that in time." And again: "As to what you say of Sir Isaac Newton's Scripture Prophecy I am inclined to think your judgement of it perfectly right. Though he was a prodigy in his way, yet I never expected great things of this kind . . . from a man who spent all his days looking through a telescope."

But in contrast to such polemic it is refreshing to catch a glimpse of Newton's natural, breezy, common-sense way of looking at thorny problems. Here is a passage quoted by Prof. Brett, giving a question asked in reference to the "homoousios" controversy: "Whether Christ sent his apostles to teach metaphysics to the unlearned common people and to their wives and children?"

Prof. Brett concludes that Newton affected religious thought in a far deeper way than by additions to the literature of theology. He destroyed "the ancient tradition of the two worlds, celestial and terrestrial," and established "the idea of law in the Universe as one and the same throughout."

To Mr. G. E. Roberts, an ex-Director of the United States Mint, is appropriately entrusted the account of Newton's handling of the delicate problem of the currency. He looks upon Newton at the time he became Warden of the Mint as a moody, unhappy bachelor, with limited means, not in the best of health, and "frankly . . . an applicant for a place in the Government. . . . Great men have not been free from some of the minor weaknesses of mankind."

But is it fair to suppose that there is no alternative explanation? The country was on the brink of a serious crisis. Newton was fully aware of the magnitude and the gravity of the task that lay before him, and of the reasons why he was selected to devise and to carry out remedial measures. Is it not possible that the high sense of public duty implied in Wellington's oft-repeated phrase, "the King's Government must be carried on," has its parallel in Newton's: "I do not love to be thought by our own people to be trifling away my time about (mathematical things), when I am about the King's business"?

It is not out of place to quote the opinion of an accomplished historian, the Regius Professor of Modern History at Cambridge. Apropos of comments made by a writer in *The Times*, Prof. G. M. Trevelyan addressed the editor as follows:

"Last of all, does your reviewer do justice to Newton's public spirit? Your reviewer almost appears to regard Newton's feeling for his post at the Mint as a mild form of snobbery. He quotes, with apparent disapproval, the noble words in which Newton declares that he cannot be disturbed by scientific inquirers 'when I should be about the King's business.' Is not this the spirit that saves nations? In an age of corrupt and self-seeking and treacherous politicians a great financier was struggling in war-time with the terrible problem of the recoinage, and called in the aid of this great mathematician and scientist to give a few years 'to the King's business.' Is not that a record for a country to be proud of? Is it not at least highly significant that the greatest poetical artist our race ever produced—Shakespeare alone excepted—practically abandoned his art for twenty years in order to devote himself to the political crises of his country, and that in the following generation the greatest of our scientific men was proud for a few years 'to be about the King's business'? And we got 'Paradise Lost' and the 'Principia' all the same."

The next contribution, by Mr. F. E. Brasch of the Washington Library of Congress, is on *Newton's first critical Disciple in the American Colonies—John Winthrop*. It contains much material that is probably new and certainly of interest to British readers. It is a curious coincidence that while writing this notice a companion is provided for Winthrop on the roll of men of eminence in America who can claim a connexion with the history of poetry in England. The recent death of Myron Herrick, ambassador in France during part of the Great War, reminds us that he was of the family of one who will be familiar to some as the author of the exquisite *Hesperides*, and to perhaps a larger body of readers as the author of that immortal song *To Anthea*. And Winthrop Mackworth Praed came of the same stock as the

American Winthrop—Praed, the laureate-elect, as Austin Dobson calls him, "justly acknowledged as supreme" among writers of *vers de société* and absent from no *Lyra Elegantiarum*. The Mackworths, too, were kinsmen of the poet Waller.

John Winthrop (1588-1649) was educated at Trinity, Cambridge. In 1629 he went out to Massachusetts, of which colony he eventually became Governor. His son John (1606-1676) was educated at Trinity College, Dublin. Young John's uncle was Sir George Downing, a notable character—"le plus grand querelleur des diplomates de son temps," said Colbert—whose grandson founded Downing College, Cambridge. Downing had his interests overseas (he was the second graduate on the roll of Harvard), and he induced his nephew to return to his father in America, where in due course John became Governor of Connecticut. He was interested in science, and Mr. Brasch tells us that he had a large correspondence with men like Napier, Boyle, Hooke, and Newton. But as Napier died in 1617, John's correspondence with him must have been of a boyish character. Winthrop gradually became the trans-Atlantic correspondent for the Royal Society. He carried a loyal address from the Colony to Charles II. in 1662, and the next year we see his name on the roll of the Royal Society. Apparently he had some repute in his own country as an astronomer. "In astronomy," says Mr. Brasch, "his knowledge was more advanced than that of the average colonist for he possessed a small telescope and made observations of unique value. It is recorded as far back as 1664 that he made the astounding discovery of the fifth satellite of Jupiter, over two hundred years before it was actually discovered at Lick Observatory." This was good observing, for this baby moon is only 100 miles in diameter. His grandson was a botanist. He became an F.R.S. in 1734, and the fortieth volume of the *R.S. Transactions* is dedicated to him. The last of the family bearing the title F.R.S. (1766) is the "first disciple" of Mr. Brasch's paper. The second edition (1713) of Newton's *Principia* reached Yale in 1714, and in that year "a child was born who was destined to carry the torch of Sir Isaac Newton to still greater heights and to maintain it there." From the age of twenty-four and for the next forty years this John Winthrop was Professor of Mathematics and Natural Philosophy at Harvard. For the part he played in the development of the intellectual life of the colonies we must refer the reader to the very interesting narrative of Mr. Brasch. "It was this work of Winthrop's which laid the solid foundation of Harvard's pre-eminence in science, and inspired the spirit of research which has made this country outstanding in the history of science."

It remains to congratulate the scientific societies connected with the arrangement of this commemorative volume on a substantial and valuable addition to the literature connected with the name of Newton.

W. J. Greenstreet.

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669. "The third caste is that of the Banians. They accustom their children at an early age to shun slothfulness, and instead of letting them go into the streets to lose their time at play, as we generally allow ours, teach them arithmetic, which they learn perfectly, using for it neither pen nor counters, but the memory alone, so that in a moment they will do a sum, however difficult it may be."—Tavernier, *ibid.* vol. ii. p. 144. [Per Mr. Puryer White.]

670. "(In Benares) there is a house which serves as a college, which the Raja Jai Singh . . . has founded for the education of the youth of good families. . . . Entering the court of this college, being curious to see it, and throwing my eyes upwards, I perceived a double gallery which ran all round it, and in the lower the two Princes were seated, accompanied by many young nobles and numerous Brahmans, who were making different figures, like those of mathematics on the ground with chalk. . . . One of the Brahmans had two globes, which the Dutch had given him, and I pointed out the position of France upon them."—Tavernier, *ibid.* vol. ii. p. 183. [Per Mr. Puryer White.]

VICTORIA BRANCH.

Report for Sessions 1927-1928.

Honorary President: Professor E. J. Nanson. *President*: Professor J. H. Michell. *Vice-Presidents*: Miss K. Gillman Jones; Mr. O. K. Picken; Mr. M. S. Sharman. *Committee*: Miss J. T. Flynn; Miss W. Waddell; Mr. C. H. Wickens. *Secretaries*: Mr. R. J. A. Barnard; Mr. J. L. Griffiths. *Treasurer*: Mr. F. W. Campbell.

Session 1927.

Five meetings in all were held during the year.

The meetings on 25th April and 20th June were devoted to a discussion on the Teaching of Arithmetic in Schools. On the first evening there was an attendance of about 50. The discussion was opened by Miss K. Gilman Jones with an address on Arithmetic Teaching in its most elementary stages. Mr. J. A. Seitz continued the discussion, speaking of more advanced work and discussing the question under three headings: (1) Syllabus and Preparation, (2) Teaching, (3) Examining. Other aspects were dealt with at the second meeting, at which Miss Kirkhope and Mr. J. L. Griffiths were the principal speakers.

On 18th July Mr. R. J. A. Barnard gave a paper on Differential Equations connecting Three Variables. He dealt with the equations:

$$(1)\ dx/P = dy/\phi = dz/r\,;$$

$$(2)\ Pdx + Qdy + Rdz = 0\,;$$

$$(3)\ Pdz/dx + \phi dz/dy = R\,;$$

and explained the geometrical relation in each case. He pointed out the unsatisfactory way in which the solutions of Partial Differential Equations were presented in the text-books, and referred to some of the erroneous statements to be found regarding the relationship of the above equations. He also doubted the necessity for the introduction of Special Solutions of P.D.E.'s, as in all the illustrations given in the text-books the solutions are obtainable as singular solutions, provided the equation is treated in the way that ordinary D.E.'s are treated in obtaining singular solutions.

On 19th September Professor Michell gave a paper on Twist in Mathematics. He started from the geometrical equation

$$d\phi/ds + \bar{\imath} = \text{rate of rotation},$$

in going along a curve, by a line in the normal plane, making an angle ϕ with the principal normal, $\bar{\imath}$ being the torsion.

Then he spoke of other cases in which an equation of the same kind occurred, as in the agyral top. He illustrated it also by the cases of a coil of rope, a piece of rubber coiled on a cylinder, congruences of straight lines, etc. He showed experiments in illustration, *e.g.* a cylindrical piece of rubber with pins inserted to represent normals, and also a model of a congruence of straight lines.

On 17th October Mr. M. H. Belz gave an account of the development of the Quantum Theory. He explained the origin of Planck's *Theory of Radiation*, and outlined the work of Wien, Rayleigh, Einstein and Debye, and showed how they led to the theories of Compton, Kramers, and Heisenberg.

Session 1928.

Five meetings were held in the year.

On 16th April Mr. W. R. Strong read a paper on Statistical Methods. He gave an account of the various mean values employed in statistics and their uses, and explained their application to statistical problems. He also dealt

with frequency curves and coefficients of correlation, dispersion, standard deviation, etc. There were about 40 members and visitors at the meeting.

On 14th May Dr. Baldwin, Government Astronomer, gave a paper on Numerical Computation by Alignment Charts. He explained their construction and the use that he had made of them at the Observatory for deducing the true coordinates of a star from the x and y coordinates on a photograph. He found the work greatly shortened by the use of the charts, and the required accuracy was obtained.

On 18th June Mr. G. Gundersen gave a paper on Complex Numbers in School Teaching. He gave a history of the introduction of unreal numbers, and discussed the arrangement of the work on it required in a school course.

At the same meeting Miss E. F. Allen gave a demonstration of the Integraph, explaining the principles of the instrument and the method of using it.

The meeting of 16th July was devoted to miscellaneous short contributions by the members.

Mr. Campbell asked for an explanation of discrepant results he obtained in a question of probability, and was satisfactorily answered.

Mr. O. K. Picken spoke of points arising in the discussion of trigonometrical limits when any unit of angle was used. He also spoke of the method of obtaining the canonical form of the equation of a central conic from the focus and directrix property.

Mr. R. J. A. Barnard read a note on the extreme values of

$$(ax^2+2bx+b)/(Ax^2+2Hx+B)$$

dealing with the condition for their existence, in the form that the roots of both denominator and numerator must be real and the roots of one must separate those of the other. He showed that this condition could be got very easily by the calculus method.

Professor Michell gave a demonstration of the stabilized pendulum. The point of support of a short pendulum was made to oscillate by means of an electric motor. The stability of the inverted position for sufficiently rapid oscillations was very clearly shown.

On 25th September Mr. Massey read a paper on the New Quantum Theory of Ideal Gases. He gave an outline of the Fernie Statistical Method and an account of the results obtained by it. R. J. A. Barnard (Hon. Sec.).

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MATHEMATICAL ASSOCIATION.

LOCAL BRANCHES.

LIST OF HONORARY SECRETARIES.

LONDON:	Miss L. A. ZELENSKY, Haberdashers' Aske's Girls' School, Creffield Road, Acton, W. 3. F. C. BOON, 49 Idmiston Road, West Norwood, S.E. 27.
NORTH WALES:	A. P. CHAPMAN, 47 Holyhead Road, Bangor.
YORKSHIRE:	A. B. OLDFIELD, The Secondary School, Pudsey, near Leeds.
BRISTOL:	G. W. HINTON, 32 Tyndalls Park Road, Bristol.
MANCHESTER AND DISTRICT:	Miss M. O. STEPHENS, Fairfield High School, Manchester. G. B. JACKSON, Secondary School, Ashton-under-Lyne.
CARDIFF:	H. A. HAYDEN, University College, Cardiff.
MIDLAND:	Miss E. M. READ, King's Norton Secondary School.
NORTH EASTERN:	Miss M. WAITE, The High School, Darlington. A. K. WILSON, Dame Allan's School, Newcastle-upon-Tyne.
SYDNEY, N.S.W.:	Miss JANET BROWN, Boys' High School, Fort Street, Sydney. H. J. MELDRUM, The Teachers' College, Sydney.
QUEENSLAND:	J. P. MCCARTHY, Boys' Grammar School, Gregory Terrace, Brisbane.
VICTORIA:	R. J. A. BARNARD, 21 Bambra Road, Caulfield. J. L. GRIFFITH, 1032 Drummond Street, North Carlton.

LONDON BRANCH.

PROGRAMME FOR 1928-29.

All meetings will be held at Bedford College, Regent's Park.
They will be on Saturday afternoons, and will begin at 3 p.m.

1929.

Feb. 2nd. Presidential Address: "Dimensions and Identity of Vector direction."—Professor ALFRED LODGE.

Feb. 23rd. Annual Business Meeting.
Discussion of Members' Questions.

Mar. 16th. Debate on Standard Form.
(Speakers: Miss PUNNETT and A. S. GRANT.)

The Secretaries will be glad to receive:

(i) Topics for discussion on February 23rd.
(ii) Names of Members willing to take part in the Debate on March 16th.

THE MATHEMATICAL ASSOCIATION.

(*An Association of Teachers and Students of Elementary Mathematics.*)

"*I hold every man a debtor to his profession; from the which as men of course do seek to receive countenance and profit, so ought they of duty to endeavour themselves, by way of amends, to be a help and an ornament thereunto.*"—BACON (Preface, *Maxims of Law*).

THE MATHEMATICAL ASSOCIATION, which was founded in 1871, as the *Association for the Improvement of Geometrical Teaching*, aims not only at the promotion of its original object, but at bringing within its purview all branches of elementary mathematics.

Its purpose is to form a strong combination of all persons who are interested in promoting good methods of teaching mathematics. The Association has already been largely successful in this direction. It has become a recognised authority in its own department, and is continuing to exert an important influence on methods of examination.

The Annual Meeting of the Association is held in January. Other Meetings are held when desired. At these Meetings papers on elementary mathematics are read and discussed.

Branches of the Association have been formed in London, Bangor, Yorkshire, Bristol, Manchester, Cardiff, the Midlands (Birmingham), New South Wales (Sydney), Queensland (Brisbane), and Victoria (Melbourne). Further information concerning these branches can be obtained from the Honorary Secretaries of the Association.

"The Mathematical Gazette" (published by Messrs. G. BELL & SONS, LTD.) is the organ of the Association. It is issued at least six times a year. The price per copy (to non-members) is usually 2s. 6d. each. The *Gazette* contains—

(1) ARTICLES, mainly on subjects within the scope of elementary mathematics;

(2) NOTES, generally with reference to shorter and more elegant methods than those in current text-books;

(3) REVIEWS, written when possible by men of eminence in the subject of which they t eat. They deal with the more important English and Foreign publications, and their aim is to dwell on the general development of the subject, as well as upon the part played therein by the book under notice;

(4) QUERIES AND ANSWERS, on mathematical topics of a general character.